Reconstruction of High Resolution Images from Movie Frames

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Abstract

This paper extends the idea of high-resolution color image reconstruction from multisensors to images from video frames. The high resolution color images are reconstructed from multiple undersampled, shifted, degraded color movie frames with approximately one pixel displacements. The weighted H_1 regularization functional is used to capture the reflectivity across color channels and the Neunmann boundary condition is also employed to reduce the boundary artifacts. Examples are given to illustrate the results by this algorithm.

1 Introduction

Electronic surveillance system has a wide-range of usage, for example to enhance security. Reconstruction of the high-resolution image from the recorded movie in this kind of systems is of great importance. In this paper, we consider the reconstruction of high-resolution images from multiple undersampled, shifted, degraded and noisy color movie frames which are obtained from a movie of a target object.

We separated the color movie frames into a set of three images in their primary color components: red, green and blue. The reconstruction of high resolution color images can be modeled as solving

$$\vec{g} = \mathcal{A}\vec{f} + \vec{\eta}, \tag{1}$$

where $\mathcal{A}$ is the reconstruction operator, $\vec{\eta}$ represents unknown Gaussian noise or measurement errors, $\vec{g}$ is the observed high resolution color image formed from the low resolution color movie frames and $\vec{f}$ is the desired high resolution color image. The observed and original color images can be expressed as

$$\vec{g} = \begin{pmatrix} g^{(r)} \\ g^{(g)} \\ g^{(b)} \end{pmatrix}, \quad \vec{f} = \begin{pmatrix} f^{(r)} \\ f^{(g)} \\ f^{(b)} \end{pmatrix},$$

where $g^{(i)}$ and $f^{(i)}$ $(i \in \{r, g, b\})$ are the observed and the original color images from the red, green and blue channels respectively.

Since the system (1) is ill-conditioned and generally not positive definite, we solve it by using a minimization and regularization technique:

$$\min_f \left\{ \sum_{i \in \{r,g,b\}} \alpha_i \left\| \sum_{j \in \{r,g,b\}} \mathcal{A}^{ij} f^{(j)} - g^{(i)} \right\|_2^2 + \mathcal{R}(f^{(r)}, f^{(g)}, f^{(b)}) \right\}. \tag{2}$$

Here the operators $\mathcal{A}^{ii}$ and $\mathcal{A}^{ij} (i \neq j)$ are the entries of $\mathcal{A}$ and represent the within-channel and the cross-channel degradation operators respectively, $\mathcal{R}$ is a functional which measures the regularity of f, and the regularization parameter α_i is to control the degree of regularity of the solution for the i-th channel.

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The main aim of this paper is to extend our results in [1] from still images to movie frames. We apply the fast and stable image processing algorithm in [1] to the color image reconstruction problems.

The outline of the paper is as follows. In Section 2, we give a mathematical formulation of the problem. We describe the acquisition of the movie frames in Section 3. In Section 4, experimental results are presented to demonstrate the effectiveness of our method.

2 The Mathematical Model

The mathematical model is the same as [1]. Although we are using movie frames rather than taking still images from a sensor array, when the movie is taken with sufficient frame rate, we can extract the movie frames with approximately the same subpixel displacement as taking still images from the sensor array by taking advantage of the movement of the target object in the movie. Therefore we can still use the model in [1], i.e., with a sensor array with $L_1 \times L_2$ sensors, each sensor has $N_1 \times N_2$ sensing elements, and the size of each sensing element is $T_1 \times T_2$. We would like to reconstruct a high-resolution image of resolution $M_1 \times M_2$ where $M_1 = L_1 \times N_1$ and $M_2 = L_2 \times N_2$. To maintain the aspect ratio of the reconstructed image, we only consider the case where $L_1 = L_2 = L$. For simplicity, we assume L is an even number.

Ideally, the sensors are shifted from each other by a value proportional to $T_1/L \times T_2/L$. For simplicity, we assume there is no imperfection of the imaging system, i.e., there are no perturbations around ideal subpixel location. Thus, for $l_1, l_2 = 0, 1, \ldots, L-1$ with $(l_1, l_2) \neq (0,0)$, the horizontal and vertical displacements $d^x_{l_1 l_2}$ and $d^y_{l_1 l_2}$ are given by $d^x_{l_1 l_2} = \frac{T_1 l_1}{L}$ and $d^y_{l_1 l_2} = \frac{T_2 l_2}{L}$.

Let $f^{(r)}, f^{(g)}$ and $f^{(b)}$ be the original scene in red, green and blue channels respectively. The high-resolution reconstruction problem can be modeled as

$$g^{(i)} = \sum_{j \in \{r,g,b\}} w_{ij} \mathcal{H} f^{(j)} + \eta^{(i)}, \quad i \in \{r,g,b\}. \tag{3}$$

Here $g^{(i)}$ is the observed high-resolution image, $\mathcal{H} f^{(j)}$ is the formation of the low-resolution images. See [1] for more details. The parameter $\eta^{(i)}$ is the noise in the i-th channel, and w_{ii} and w_{ij} $(i \neq j)$ are the within-channel and the cross-channel degradation parameters. We note that

$$w_{ij} \geq 0, \quad i,j \in \{r,g,b\} \quad \text{and} \sum_{j=r,g,b} w_{ij} = 1, \quad i \in \{r,g,b\}.$$

2.1 The Discrete Model

For $i \in \{r,g,b\}$, let $\mathbf{g}^{(i)}$ and $\mathbf{f}^{(i)}$ be the respectively the discretization of $g^{(i)}$ and $f^{(i)}$ using a column by column ordering. Let

$$\mathbf{g} = [\mathbf{g}^{(r)} \quad \mathbf{g}^{(g)} \quad \mathbf{g}^{(b)}]^t \quad \text{and} \quad \mathbf{f} = [\mathbf{f}^{(r)} \quad \mathbf{f}^{(g)} \quad \mathbf{f}^{(b)}]^t.$$

Under the Neunmann boundary condition assumption, the resulting matrices, denoted by $H^x_{l_1 l_2}$ and $H^y_{l_1 l_2}$ have a Toeplitz-plus-Hankel structure:

$$\mathbf{H}^x_{l_1 l_2} = \frac{1}{L} \begin{pmatrix} 1 & \cdots & 1 & h^{x+}_{l_1 l_2} & & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \\ 1 & \ddots & \ddots & \ddots & \ddots & h^{x+}_{l_1 l_2} \\ h^{x-}_{l_1 l_2} & \ddots & \ddots & \ddots & \ddots & 1 \\ & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & & h^{x-}_{l_1 l_2} & 1 & \cdots & 1 \end{pmatrix} + \frac{1}{L} \begin{pmatrix} 1 & \cdots & 1 & h^{x-}_{l_1 l_2} & & 0 \\ \vdots & \iddots & \iddots & & & \\ 1 & \iddots & & & & h^{x+}_{l_1 l_2} \\ h^{x-}_{l_1 l_2} & & & & \iddots & 1 \\ & & & \iddots & \iddots & \vdots \\ 0 & & h^{x+}_{l_1 l_2} & 1 & \cdots & 1 \end{pmatrix}$$

and $\mathbf{H}^y_{l_1 l_2}$ is defined similarly. The degradation matrix corresponding to the (l_1, l_2)-th sensor under the Neunmann boundary condition is given by $\mathbf{H}_{l_1 l_2} = \mathbf{H}^x_{l_1 l_2} \otimes \mathbf{H}^y_{l_1 l_2}$. The degradation matrix for the whole sensor array is made up of degradation matrices from each sensor:

$$\mathbf{H}_L = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \mathbf{D}_{l_1 l_2} \mathbf{H}_{l_1 l_2}, \quad i, j \in \{r, g, b\}.$$

Here $\mathbf{D}_{l_1 l_2}$ are diagonal matrices with diagonal elements equal to 1 if the corresponding component of the observed low resolution image comes from the (l_1, l_2)-th sensor and zero otherwise, see [2] for more details. From (3), we see that we have the same matrix $\mathbf{H}_L$ within the channels and across the channels, the overall degradation matrix is given by

$$\mathbf{A}_L = \begin{pmatrix} w_{rr} & w_{rg} & w_{rb} \\ w_{gr} & w_{gg} & w_{gb} \\ w_{br} & w_{bg} & w_{bb} \end{pmatrix} \otimes \mathbf{H}_L \equiv \mathbf{W} \otimes \mathbf{H}_L. \tag{4}$$

2.2 Regularization

We use the following weighted discrete Laplacian matrix $\mathbf{R}$ proposed by Galatsanos et al. in [3] with $\|\tilde{\mathbf{f}^{(i)}}\|_2$ replaced by $\|\mathbf{g}^{(i)}\|_2$ as the regularization matrix.

$$\mathbf{R} = \begin{pmatrix} 2 & -\frac{\|g^{(r)}\|_2}{\|g^{(g)}\|_2} & -\frac{\|g^{(r)}\|_2}{\|g^{(b)}\|_2} \\ -\frac{\|g^{(g)}\|_2}{\|g^{(r)}\|_2} & 2 & -\frac{\|g^{(g)}\|_2}{\|g^{(b)}\|_2} \\ -\frac{\|g^{(b)}\|_2}{\|g^{(r)}\|_2} & -\frac{\|g^{(b)}\|_2}{\|g^{(g)}\|_2} & 2 \end{pmatrix} \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{L} = \mathbf{S} \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{L}, \tag{5}$$

where $\mathbf{L}$ is the 2-dimensional discrete Laplacian matrix with the Neunmann boundary condition.

Using Tikhonov regularization in (2), our discretization problem becomes:

$$(\mathbf{A}_L^t \Upsilon \mathbf{A}_L + \mathbf{R}^t \mathbf{R}) \mathbf{f} = \mathbf{A}_L^t \Upsilon \mathbf{g}, \tag{6}$$

where

$$\Upsilon = \begin{pmatrix} \alpha_r \mathbf{I} & 0 & 0 \\ 0 & \alpha_g \mathbf{I} & 0 \\ 0 & 0 & \alpha_b \mathbf{I} \end{pmatrix} = \begin{pmatrix} \alpha_r & 0 & 0 \\ 0 & \alpha_g & 0 \\ 0 & a & \alpha_b \end{pmatrix} \otimes \mathbf{I} \equiv \Omega \otimes \mathbf{I}, \tag{7}$$

and α_r, α_g and α_b are the regularization parameters which are assumed to be positive scalars. According to (5), (7), and when the Neunmann boundary condition is used for both $\mathbf{H}_L$ and $\mathbf{L}$, (6) can be simplified to

$$[\mathbf{W}^t \Omega \mathbf{W} \otimes \Lambda^2 + (\mathbf{S} \otimes \mathbf{I} + \mathbf{I} \otimes \Sigma)^t (\mathbf{S} \otimes \mathbf{I} + \mathbf{I} \otimes \Sigma)] \tilde{\mathbf{f}} = (\mathbf{W}^t \Omega \otimes \Lambda) \tilde{\mathbf{g}}, \tag{8}$$

where Λ and Σ are diagonal matrices with diagonal entries given by the eigenvalues of $\mathbf{H}_L$ and $\mathbf{L}$ respectively, $\tilde{\mathbf{f}} = (\mathbf{I} \otimes \mathbf{C}_{M_1} \otimes \mathbf{C}_{M_2}) \mathbf{f}$ and $\tilde{\mathbf{g}} = (\mathbf{I} \otimes \mathbf{C}_{M_1} \otimes \mathbf{C}_{M_2}) \mathbf{g}$. The system in (8) is a block-diagonalized system of $M_1 M_2$ decoupled systems. The vector $\tilde{\mathbf{f}}$ can be computed by solving a set of $M_1 M_2$ decoupled 3-by-3 matrix equations.

3 Acquisition of Low-Resolution Movie Frames

In this section, we discuss the case $L = 2$, $T_1 = T_2 = 1$ here. We would like to get four low-resolution movie frames with subpixel displacements. First we fix our target object and take movie of the target object by circular motion of the camera, see Figure 1. Next we have to get the four low-resolution movie frames from the recorded movie. The criteria for choosing four movie frames is that they have to be different in each other in terms of 2-norm and the target object should be located in four different positions in the captured movie frames, see Figure 2.

After that, we have to crop the target object out of the movie frames and align them so that they are shifted from each other by approximately one pixel, see Figure 3. Note that the cropped images should all have the same size. These four cropped images serve as the input images for our algorithm.

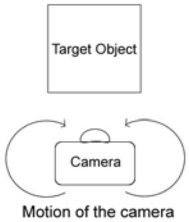

Figure 1

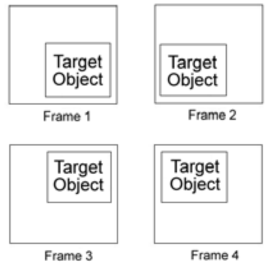

Figure 2

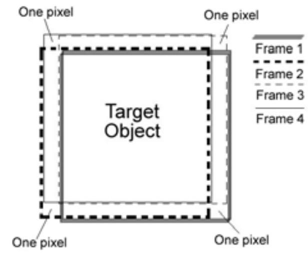

Figure 3

4 Experimental Results

In this section, we illustrate our method by recording a target object in a movie. We tried the degradation matrix in our examples with $w_{rr} = w_{gg} = w_{bb} = 0.8$ and $w_{ij} = 0.1$ for $i \neq j$ in (4). In the tests, we used the same regularization parameter for each channel, i.e., $\alpha_r = \alpha_g = \alpha_b = \alpha = 70$. One of the cropped movie frames, the corresponding observed high-resolution image, and the reconstructed high-resolution image are shown in Figures 4, 5 and 6 respectively. For the color version of Figures 4–6, please refer to [4].

Figure 4

Figure 5

Figure 6

References

[1] M. Ng, W. Kwan, and R. Chan, *A Fast Algorithm for High-Resolution Color Image Reconstruction with Multisensors*, Lecture Notes in Computer Science, 1988 (2001), 615–627.

[2] N. Bose and K. Boo, *High-resolution image reconstruction with multisensors*, International Journal of Imaging Systems and Technology, 9 (1988), 294–304.

[3] N. Galatsanos, A. Katsaggelosm, R. Chin, and A. Hillery, *Least Squares Restoration of Multichannel Images*, IEEE Trans. Signal Processing, 39 (1991), 2237–2252.

[4] R. Chan and R. Ling, *Reconstruction of High-Resolution Image from Movie Frames*, CUHK Math. Dept. Research Report #2002-30. WWW: ftp://ftp.math.cuhk.edu.hk/report/2002-30.ps.Z