德艺双馨—庆祝刘家成教授荣休

Xinhan Dong, Hunan Normal University

刘家成教授是一代数学宗师, 他求学并成功成名于美国, 却为华人数学界培养了一大批中坚力量. 他人生的过去七十年是光辉灿烂的, 其中一部分的荣耀反映在他所工作的香港中文大学以及他所著作的大量深刻而美妙的学术文章中: 他的研究工作涵盖了泛函分析, 调和分析, 概率论, 测度论, 分形几何, 复分析等广泛的数学领域: 研究工作之外, 他在公共服务方面贡献了远见卓识的智慧和富有成效的工作, 使得香港中文大学的数学系从此美誉全球, 成为数学学习和研究的圣地. 愿他荣休之后能一往如故, 享受从心所欲不逾矩的时光.

Some recent progress on spectrality of some fractal measures

Xiao-Ye Fu, Central China Normal University

We review some recent progress on the spectral property of some fractal measures which include self-similar measures generated by the pair $(b, \mathcal{D})$ with $1 < b \in \mathbb{R}$ and $\mathcal{D}$ being a (modulo)-strict product form, a class of Moran measures in the form of $\mu = \delta_{b_1,\mathcal{D}_1} * \delta_{b_1 b_2,\mathcal{D}_2} * \dots$, where $b_i > 1$ and $\mathcal{D}_i = \{0, 1, \dots q_i - 1\}$ and a class of Siepinski-type measures in $\mathbb{R}^2$.

The modulus of N-dimensional U-convexity and some geometric properties in Banach spaces

Ji Gao, Community College of Philadelphia

Inspired by the concept of U-space introduced by Lau, in this talk, we introduce the modulus of n-dimensional U-convexity which simultaneously generalizes modulus of n-dimensional uniform convexity due to Kirk and modulus of U-convexity due to Gao. The properties of the modulus are investigated and the relationships between this modulus and other geometric properties of Banach spaces are studied. Some results on fixed point theory for non-expansive mappings and normal structure in Banach spaces are improved.

This talk is based on a joint research with Satit Saujung, Department of Mathematics, Khon Kaen University, Thailand.

Dirichlet forms and critical exponents on fractals

Qingsong Gu, Memorial university

Let $B^{\sigma}_{2,\infty}$ denote the Besov space defined on a compact set $K \subset \mathbb{R}^d$ which is equipped with an α-regular measure μ. The *critical exponent* σ^* is the supremum of the σ such that $B^{\sigma}_{2,\infty} \cap C(K)$ is dense in $C(K)$. It is well-known that for many standard self-similar sets K, $B^{\sigma^*}_{2,\infty}$ are the domain of some local regular Dirichlet forms. In this talk, I will explain a new situation we have explored that the underlying fractal sets admit inhomogeneous resistance scalings, which yield two types of critical exponents. We developed a general theory of this on the p.c.f. sets. Our emphasis is on two asymmetric p.c.f. sets that are constructed. We use them to illustrate and examine the theory, the function properties of the associated Besov spaces at the critical exponents, and also the Dirichlet forms on these fractals. This is a joint work with Ka-Sing Lau.

The Large-N Limit of the q-Segal-Bargmann Transform

Ching Wei Ho, University of California, San Diego

Sniady constructed a random matrix model which has a limiting noncommutative distribution of the q-Gaussian distribution. In this talk, I will introduce the Segal-Bargmann transform in the classical case and on the Sniady random matrix model. Then we will construct the q-Segal-Bargmann transform by means of operator algebra. Finally I will describe what it is meant by the Segal-Bargmann transform on the Sniady random matrix model converges to the q-Segal-Bargmann transform in L^2 sense.

Joint work with Ka-Sing Lau

Jiaxin Hu, Tsinghua University

Five years ago in 2013, I gave a talk titled "Joint work with Ka-Sing Lau", on the occasion of Ka-Sing Lau's 65th birthday held in Guang Zhou. Since then, we continue to work together on heat kernel estimates on metric spaces. In this talk, I will present what we have done in this challenging direction since the year 2013.

Asymptotics of signed Bernoulli convolutions associated with the golden ratio

Tian-you Hu, University of Wisconsin-Green Bay

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables each taking values 1 and -1 with weights $1/2$ and $-1/2$, respectively. For $0 < \rho < 1$, define a random variable

$$S_n = S_n(\rho) = \sum_{j=1}^{n} \rho^j X_j.$$

Let $\mu_n = \mu_n(\rho)$ be the discrete measure induced by S_n. It is easy to verify that μ_n converges weakly to a null measure μ for any $0 < \rho < 1$ as $n \to \infty$.

Let $\|\mu_n\|$ be the total variation of μ_n. Then $\|\mu_n\|$ converges to the regular Bernoulli convolution if $0 < \rho \leq 1/2$. But, if there is cancellation of weights in the iteration, or equivalently, if the total variation satisfies $\|\mu_n\| < 1$ for some n, then by Young's convolution inequality $\|\mu_n\|$ must decay at least exponentially as $n \to \infty$. It is then of interest to determine the exact rate of decay of $\|\mu_n\|$ in such situation.

In this talk we study the case where $\rho^{-1} = \frac{1+\sqrt{5}}{2}$ is the golden ratio satisfying $1 = \rho + \rho^2$. We show that there exists a constant $C > 0$ such that

$$\|\mu_n\| \sim C \left(\frac{\lambda}{2}\right)^n \approx C(0.771844\cdots)^n, \text{ as } n \to \infty$$

where $\lambda \in (1, 2)$ is the only real root of the equation $x^3 = 2x^2 - 2x + 2$.

This is joint work with Xianghong Chen.

On exceptional integral self-affine sets

Ibrahim Kirat, Istanbul Technical University

We consider an integral self-affine set F, which is generated by an $n \times n$ integer expanding matrix T (not necessarily a similitude) and a finite set $A \subset \mathbb{R}^n$ of integer vectors so that $F = T^{-1}(F + A)$. The estimation of the Hausdorff dimension or the Minkowski (box) dimension of F is of considerable interest. We call F an exceptional self-affine set if its Hausdorff dimension is less than its affinity dimension (or the singular value dimension). In this talk, we consider the problem of determining such sets and discuss it on a couple of classes of exceptional sets. We shall present planar numerical examples.

Random walks and induced Dirichlet forms on compact spaces of homogeneous type

Shilei Kong, Universität Bielefeld

We extend our study of random walks and induced Dirichlet forms on self-similar sets to compact spaces of homogeneous type. A partition system on such a space brings a hyperbolic structure, on which we introduce a class of reversible random walks with constant return ratios. In this talk we summarize some of the results including the identification of the boundaries, the estimations of the Martin kernel and Naïm kernel, and a criterion for the critical exponents of the induced Dirichlet forms. This is a joint work with Ka-Sing Lau and Ting-Kam Leonard Wong.

Dispersion relations of periodic quantum graphs associated with Archimedean tiling

Chun-Kong Law, National Sun Yat-sen University

We study the periodic spectrum of some differential operators, in particular the Schrödinger operator acting on infinite polygonal graphs. Using Floquet-Bloch theory, we derive and analyze on the dispersion relations of the periodic quantum graph generated by triangles and rectangles. The analytic variety, also called Bloch variety, gives the spectrum of the differential operators. Furthermore, it is well known that there are 11 types of Archimedean tilings in the plane. We take several of them. Through a systematical characteristic function method, we are able to derive the dispersion relation on the graphs formed by these tilings. We note that these dispersion relations are surprisingly simple, making it possible for further analysis.

An L^2 identity and pinned distance problem

Bochen Liu, Bar-Ilan University

Given a measure on a subset of Euclidean spaces. The L^2 spherical averages of the Fourier transform of this measure was originally used to attack Falconer distance conjecture, via Mattila's integral. In this talk, we will consider pinned distance problem, a stronger version of Falconer distance problem, and show that spherical averages imply the same dimensional threshold on both problems. In particular, with the best known spherical averaging estimates, we improve a result of Peres and Schlag on pinned distance problem significantly.

The idea is to reduce the pinned distance problem to an integral where spherical averages apply. The key new ingredient is an identity between square functions. Invariant measures on orthogonal groups plays an important role.

Spectral dimension of Laplacians defined by measures with overlaps and application to heat kernel estimates.

Sze-Man Ngai, Georgia Southern University and Hunan Normal University

We report some results on the spectral dimension of fractal Laplacians defined by one-dimensional self-similar measures with overlaps. As an application, we show that spectral dimension plays an important role in obtaining heat kernel estimates for the associated Laplacians. Some of this work is joint with Qingsong Gu, Jiaxin Hu, Wei Tang and Yuanyuan Xie.

20 years of fractal geometry with CUHK: a personal point of view

Eric Olivier, Universite D'Aix-Marseille

After my Ph. D. Thesis (in the late 90's) I was invited in by Ka-Sing Lau for a post doctoral position in the Math. Department of the CUHK (located at the Lady Shaw Building). At that time, my research was concerned with the question of phase transitions in the multifractal analysis of the so-called weak-Gibbs measures. The main mathematical program of this post-doc – as discussed with Ka-Sing Lau – was concerned with remaining difficult questions about a well known parametric family of multifractal measures called "Bernoulli Convolutions", one of them (the Erdös measure) being the Bernoulli Convolution associated with the Golden Number. The main lines of the – past and recent – mathematical developments of this question are well known and several review papers give detailed accounts of this story. In this talk, I will present a personal point of view, with an emphasis on how my work with Ka-Sing Lau and his collaborators, has contributed – on a period of 20 years – to shed lights on some interesting fractal and multifractal phenomenons.

Fourier Dimensions and Salem Sets: from Kahane to some Recent's

Narn-Rueih Shieh, National Taiwan University

Let μ be a finite Borel measure in $\mathbb{R}^d$ which support is a fractal-like set E. In his classic 1985 book, J.P. Kahane (1926-2017) proposed to study the decay rate of the Fourier transform of μ, and coined the notions of Fourier dimension and Salem set. The notions get much attentions in recently years, notably I. Laba's 2014 ICM Lecture and P. Mattila's 2015 CUP Monograph (remark: it seems Laba more on the measure, while Kahane more on the set). In this talk, we discuss some recent works on this emerging topic, in the context of Gaussian Random Fields; refer to a previous paper in Studia Math 2006 (joint work with Y. Xiao).

On Multiplicative Tilings

Yang Wang, Hong Kong University of Science and Technology

Tiling is a classical area of mathematics that has been well studied since ancient time. The classical problem involves using a finitely many shapes to cover up the Euclidean space without overlapping. A special case is translational tiling, where the tiles can only be translated, but not rotated or reflected. Here we consider multiplicative tilings, where we have one tile T and a discrete set A, such that the sets $\{aT : a \in A\}$ form a tiling of the real line. We study both the structure of the set of multiples A and the structure of the tile . We prove strong results in both cases. These results are somewhat analogous to the known results about the structure of translational tiling of the real line. There is, however, an extra layer of complexity due to the presence of sign in the sets A and T, which makes multiplicative tiling closely related to translational tiling on the larger group $\mathbb{Z}^2 \times \mathbb{R}$.

Sums of orthomorphisms of continuous functions.

Ngai-Ching Wong, National Sun Yat-sen University

A bounded linear operator T between continuous functions is called an orthomorphism if it is disjointness preserving, i.e., $TfTg = 0$ whenever $fg = 0$.

We call T an n-orthomorphism if it is n-disjointness preserving, i.e., $TfTg \cdots Th = 0$ whenever $f, g, \ldots, h$ are arbitrary n functions which are pairwise disjoint.

It is clear that a sum of n orthomorphisms is an n-orthomorphism. But a counter example shows that the converse does not hold. In this talk, we study the question of how to write an n-orthomorphism as a sum of n orthomorphisms approximately.

It is a joint work with Ching-Jou Liao (HK Baptist U.) and Jung-Hui Liu (Sanming U.).