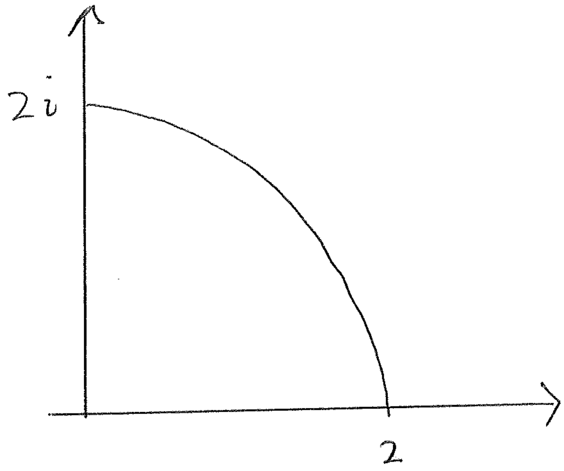
① $\left| \int_C \frac{z+5}{z^2-1} \, dz \right|$



$\leq \int_C \left| \frac{z+5}{z^2-1} \right| dz$

$\leq \int_C \frac{|z|+5}{|z|^2-1} \, dz$

$= \frac{2\pi(2)}{4} \cdot \frac{2+5}{4-1} = \frac{7\pi}{3}$

② On C, $z = 1 + 2e^{i\theta}$ with $\theta \in [-\pi, \pi)$.

$dz = 2e^{i\theta} i \, d\theta$

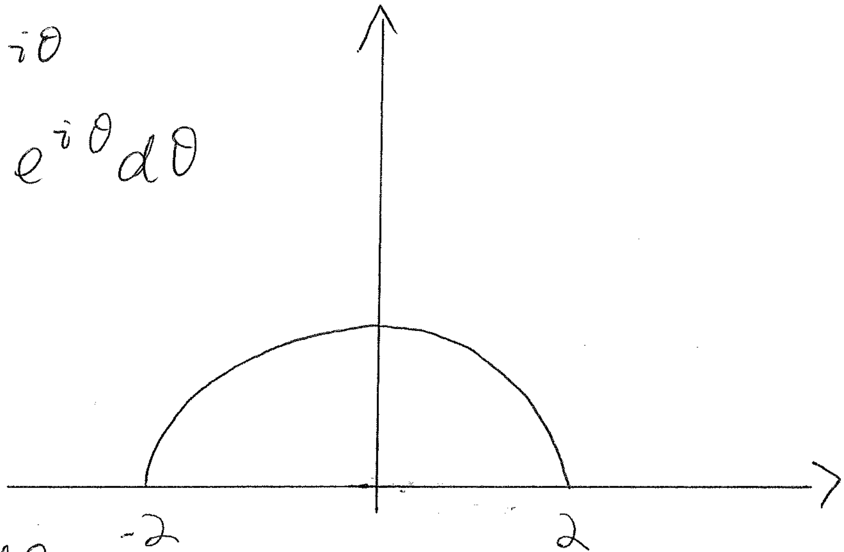
$\int_C \frac{z \, dz}{z-1} = \int_{-\pi}^{\pi} \frac{1+2e^{i\theta}}{2e^{i\theta}} 2e^{i\theta} i \, d\theta.$

$= \int_{-\pi}^{\pi} i + 2ie^{i\theta} \, d\theta$

$= 2\pi i$

(3a) On C, $z = 2e^{i\theta}$

$dz = 2ie^{i\theta}d\theta$



$$\int_C \frac{z+2}{z}\,dz$$

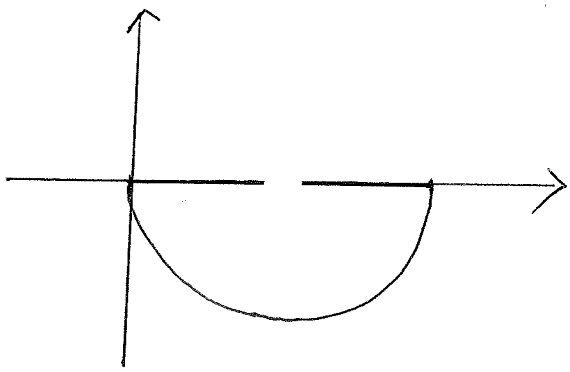
$$= \int_0^{\pi} \frac{2e^{i\theta}+2}{2e^{i\theta}}\, 2ie^{i\theta}d\theta$$

$$= \int_0^{\pi} 2ie^{i\theta} + 2i\; d\theta$$

$$= 2(e^{i\pi}-1) + 2i\pi = -4 + 2i\pi$$

(3b) on C, $z = 1 + e^{i\theta}$

$dz = ie^{i\theta}d\theta$



on line segment, $z = x$

$dz = dx$

$$\int_C z-1\,dz = \int_{\pi}^{2\pi} e^{i\theta}\, ie^{i\theta}d\theta + \int_2^0 x-1\,dx$$

$$= \frac{1}{2}\left[e^{2i\theta}\right]_{\pi}^{2\pi} + \left[\frac{x^2}{2} - x\right]_2^0$$

$$= 0$$