(1)

let $x = r\cos\theta$, $y = r\sin\theta$

$$\partial_r u = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= \frac{\partial u}{\partial x} \cos\theta + \frac{\partial u}{\partial y} \sin\theta$$

$$= \frac{1}{r}\left(\frac{\partial u}{\partial x} x + \frac{\partial u}{\partial y} y\right)$$

$$\partial_\theta v = \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$= -\frac{\partial v}{\partial x} r\sin\theta + \frac{\partial v}{\partial y} \cdot r\cos\theta$$

$$= -\frac{\partial v}{\partial x} y + \frac{\partial v}{\partial y} x$$

$$= \frac{\partial u}{\partial y} y + \frac{\partial u}{\partial x} x = r\partial_r u$$

....

OR $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\frac{y}{x}$

$\partial_x u = \; ....$

② ⓐ If $f = u + iv = x^2 - y^2 + i(2xy + y^2)$

$\partial_x u = 2x \qquad \partial_x v = 2y$

$\partial_y u = -2y \qquad \partial_y v = 2y + 2x$

~~[illegible]~~ since $\partial_x u, \partial_x v, \partial_y u, \partial_y v$ are continuous in $\mathbb{C}$, then the derivative $f'(z)$ only exist when $y = 0$. (Not analytic).

ⓑ $f = u + iv = (x + iy)y$

$\partial_x u = y \qquad \partial_x v = 0$

$\partial_y u = x \qquad \partial_y v = 2y$

..... continuous in $\mathbb{C}$, exist when $x = y = 0$.

3

$$\partial_x u = \partial_y v$$

$$\partial_y v = y$$

$$\int \partial_y v \, dy = \int y \, dy$$

$$v = y^2/2 + h(x)$$

$$\partial_x v = h'(x) = -\partial_y u = -x$$

$$h = -x^2/2 + c.$$

$$\Rightarrow f = xy + i\left(y^2/2 - x^2/2 + c \right)$$