MATH1520AB 2021-22 Tutorial 4 (week 6)

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1. Let $y = f(x)$ be a differentiable function of x. Find $\frac{dy}{dx}$ as a function of x and y, if y satisfies the equation $\tan(x^2y^4) = x + y^2$.

Answer.

$$\begin{aligned} \tan(x^2y^4) &= x + y^2 \\ (2xy^4 + 4x^2y^3\frac{dy}{dx})\sec^2(x^2y^4) &= 1 + 2y\frac{dy}{dx} \\ (4x^2y^3\sec^2(x^2y^4) - 2y)\frac{dy}{dx} &= 1 - 2xy^4\sec^2(x^2y^4) \\ \frac{dy}{dx} &= \frac{1 - 2xy^4\sec^2(x^2y^4)}{4x^2y^3\sec^2(x^2y^4) - 2y} \end{aligned}$$

2. Evaluate the following limits.

(a) $\lim_{x\to\infty} \frac{\ln(3x)}{x^2}$

(b) $\lim_{x\to 0} \frac{\sin(2x) + 7x^2 - 2x}{x^2(x+1)^2}$

Answer.

(a) $\lim_{x\to\infty} \frac{\ln(3x)}{x^2} = \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$

(b) $\lim_{x\to 0} \frac{\sin(2x) + 7x^2 - 2x}{x^2(x+1)^2} = \lim_{x\to 0} \frac{\sin(2x) + 7x^2 - 2x}{x^4 + 2x^3 + x^2} = \lim_{x\to 0} \frac{2\cos(2x) + 14x - 2}{4x^3 + 6x^2 + 2x} = \lim_{x\to 0} \frac{-4\sin(2x) + 14}{12x^2 + 12x + 2} = 7$

3. Let $f(x) = x^4 - 14x^2 + 24x$. Find all its relative maxima and relative minima.

Answer.

Since $f'(x) = 4x^3 - 28x + 24 = 4(x-1)(x-2)(x+3)$, the critical numbers are solutions of $f'(x) = 0$, i.e. $x = -3$ or $x = 1$ or $x = 2$.

x	$(-\infty, -3)$	-3	$(-3, 1)$	1	$(1, 2)$	2	$(2, \infty)$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

Relative minimum: $(-3, -117)$ and $(2, 8)$

Relative maximum: $(1, 11)$