lectures 9 and 10.

Three integrals are introduced.

Type 1. $w(t) = w_1(t) + i w_2(t)$

$$\therefore \int_a^b w(t)\,dt \triangleq \int_a^b w_1(t)\,dt + i \int_a^b w_2(t)\,dt.$$

This integral is a straight forward generalization of integral in single variable calculus.

$\therefore$ if $F(t) = F_1(t) + i F_2(t)$ s.t. $F_1'(t) = f_1(t)$, $F_2'(t) = f_2(t)$

$$\therefore \int_a^b f_1(t) + i f_2(t)\,dt = \int_a^b F_1' + i F_2'\,dt$$

$$= F_1\Big|_a^b + i F_2\Big|_a^b$$

Fundamental Thm of Calculus holds

if $[f(g(t))]' = f'(g(t))\, g'(t)$.

then $\int_a^b f'(g(t))\, g'(t)\,dt = f(g(t))\Big|_a^b$.

Type 2. $f(z)$ Complex. function. $\gamma(t)$ a parametrization of a Curve C.

$$\int_C f(z)\,dz \triangleq \int_a^b f(\gamma(t))\,\gamma'(t)\,dt$$

Here $a < b$.

Type 3. Absolute integral.

$$\int_C f(z)\,|dz| \triangleq \int_a^b f(\gamma(t))\,|\gamma'(t)|\,dt.$$

This integral is indepent of direction only on C.

- $$\left|\int_C f(z)\,dz\right| \le \int_C |f(z)|\cdot|dz|$$

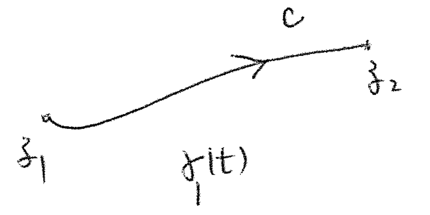
- $$|dz| = |\gamma'(t)|\,dt = \sqrt{(\gamma_1')^2+(\gamma_2')^2}\;dt$$

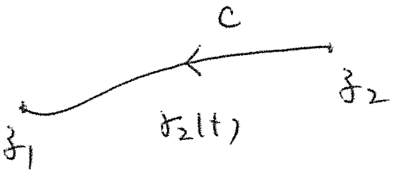
$\uparrow$

arc-differentiation

$$\therefore \int_C |dz| = \text{Length of } C.$$

Rk: This integral depends on direction of curve.





then $\displaystyle\int_{\substack{c \text{ parametrized} \\ \text{by } \gamma_1}} f(z)\,dz = -\int_{\substack{c \text{ parametrized} \\ \text{by } \gamma_2}} f(z)\,dz.$

$\therefore$ In complex analysis, curve c always means a directional curve.

if γ_1 and γ_2 give same directional curve c.

then $\displaystyle\int_{\gamma_1} f(z)\,dz = \int_{\gamma_2} f(z)\,dz$

More attention should be paid if $f(z)$ depends on choice of branch.

ex: $f(z) = z^{-1+i}$ $\quad (-\pi < \operatorname{Arg} z \le \pi)$.

$z = e^{i\theta}$. $\quad 0 < \theta < 2\pi$.