THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 9 (Due on)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P196-197

8 Rederive the Maclaurin series (4) in Sec. 64 for the function $f(z) = \cos z$ by

a using the definition
$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$
in Sec.37 and appealing to the Maclaurin series (2) in Sec. 64;

b showing that
$$f^{(2n)}(0) = (-1)^n \quad \text{and} \quad f^{(2n+1)}(0) = 0 \quad (n = 0, 1, 2, ...)$$

9 Use representation (3). Sec. 64. for $\sin z$ to write the Maclaurin series for the function
$$f(z) = \sin(z^2),$$
and point out how it follows that
$$f^{(4n)}(0) = 0 \quad \text{and} \quad f^{(2n+1)}(0) = 0 \quad (n = 0, 1, 2, ...)$$

10 Derive the expansions

a $\dfrac{\sinh z}{z^2} = \dfrac{1}{z} + \sum_{n=0}^{\infty} \dfrac{z^{2n+1}}{(2n+3)!} \quad (0 < |z| < \infty);$

b $\dfrac{\sin(z^2)}{z^4} = \dfrac{1}{z^2} - \dfrac{z^2}{3!} + \dfrac{z^6}{5!} - \dfrac{z^{10}}{7!} + ... \quad (0 < |z| < \infty).$

11 Show that when $0 < |z| < 4$,
$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{(4^{n+2})}$$

P205-206

1 Find the Laurent series that represents the function
$$f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$$
in the domain $0 < |z| < \infty$.

2 Find a representation for the function

$$f(z) = \frac{1}{1+z} = \frac{1}{z} \cdot \frac{1}{1+1/z}$$

in negative powers of z that is valid when $1 < |z| < \infty$.

5 The function

$$f(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}.$$

which has the two singular points $z = 1$ and $z = 2$, is analytic in the domains

$$D_1 : |z| < 1, \quad D_2 : 1 < |z| < 2, \quad D_3 : 2 < |z| < \infty.$$

Find the series representation in powers of z for $f(z)$ in each of those domains.

6 Show that when $0 < |z-1| < 2$,

$$\frac{z}{(z-1)(z-3)} = -3 \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}$$

7 a Let a denote a real number, where $-1 < a < 1$. and derive the Laurent series representation

$$\frac{a}{z-a} = \sum_{n=1}^{\infty} \frac{a^n}{z^n} \quad (|a| < |z| < \infty).$$

b After writing $z = e^{i\theta}$ in the equation obtained in part (a), equate real parts and then imaginary parts on each side of the result to derive the summation formulas

$$\sum_{1}^{\infty} a^n \cos n\theta = \frac{a\cos\theta - a^2}{1 - 2a\cos\theta + a^2} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{a\sin\theta}{1 - 2a\cos\theta + a^2}$$

where $-1 < a < 1$.