THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2230 Tutorial 6

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Theorem 1. *(Cauchy-Goursat theorem) If $f(z)$ is analytic at all points interior to and on a simple closed contour C (the closure of the bounded component divided by the contour), then*

$$\int_C f(z)dz = 0$$

Remark: You may use Cauchy Riemann equation to check the analyticity. Or you may just see if the function is composed by some elementary analytic functions. (polynomial, trigonometric function, exponential function...)

Theorem 2. *Suppose that*

1. *C is simply closed contour in counterclockwise direction;*
2. *C_k(n=1,..,n) are simply closed contour interior to C, all in clockwise direction, that are disjoint and whose interiors have no common points.*

If f is analytic on all of the contour C and C_k and throughout the multiply connected domain consisting of the points inside C and exterior to each C_k, then

$$\int_C f dz + \sum_1^n \int_{C_k} f dz = 0$$

Remark : You should draw a diagram about it.
Remark : $\int_C f dz = -\int_{-C} f dz$ where C is in counterclockwise direction and $-C$ is in clockwise direction.
Remark : You can replace the contour C with a circle or other "simple" contour in most of the case.

Exercise: 1 Use theorem 1 to show that the integrals are zero along the contour $|z| = 1$
(a) $\displaystyle\int_C \frac{dz}{z^2 + 2z + 2}$ (b) $\displaystyle\int_C Log(z + 2)dz$.