HW 7

P 147

① z^n has an antiderivative $z^{n+1}/n+1$ in $\mathbb{C}$, Thus the result follows.

② ⓐ $$\int_0^{1+i} z^2\,dz = \left[\frac{z^3}{3}\right]_0^{1+i} = \frac{1}{3}(1+i)^3 = \frac{\sqrt{2}^3}{3} e^{3\pi/4}$$

$$= \frac{2\sqrt{2}}{3}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right) = \frac{2}{3}(-1+i)$$

ⓑ $$\int_0^{\pi+2i} \cos\frac{z}{2}\,dz = 2\left[\sin\frac{z}{2}\right]_0^{\pi+2i} = 2\left(\sin\frac{\pi+2i}{2}\right)$$

$$= 2\left(\frac{e^{i(\pi+2i)/2} - e^{-i(\pi+2i)/2}}{2i}\right) = \left(\frac{ie^{-1} - \frac{1}{i}e}{i}\right) = e + e^{-1}$$

ⓒ $$\int_1^3 (z-2)^3\,dz = \left[\frac{(z-2)^4}{4}\right]_1^3 = \frac{1}{4}(1-1) = 0$$

③ Since $(z-z_0)^{n-1}$ has antiderivative $\frac{(z-z_0)^n}{n}$ ~~when~~ for $n = \pm 1, \pm 2 \ldots$, then by thm in Sect. 48, the result follow.

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$$\int_{-1}^{1} z^i dz = \int_{-1}^{1} e^{i\log z} dz = \int_{-1}^{1} e^{i\log x} dx$$

$$= \int_0^1 e^{i\log x} dx + \int_{-1}^{0} e^{i\log x} dx$$

$$\int_{-1}^{0} e^{i\log x} dx = \int_{1}^{0} e^{i\log(-x)} d(-x) \quad (\text{replace } x \text{ by } -x)$$

$$= \int_0^1 e^{i(\log(-1) + \log x)} dx$$

$$= \int_0^1 e^{-\pi} e^{i\log x} dx$$

$$\int_{-1}^{1} z^i dz = \cancel{2\int_0^1 e^{i\log x} dx + \int_0^1 e^{-\pi}}$$

$$= (e^{-\pi} + 1) \int_0^1 e^{i\log x} dx$$

let $y = \log x$

$$\int_{-1}^{1} z^i dz = (e^{-\pi} + 1) \int_{-\infty}^{0} e^{iy+y} dy$$

$$= (e^{-\pi} + 1)\left(\frac{1-i}{2}\right)$$

P159 3 We can choose R large enough s.t. the rectangle is strictly contained inside the circle $\{|z-z_0| = R\}$. Or, choosing R small enough s.t. the circle is strictly contained inside the rectangle. Then it is done by $\int_{C_0} (z-z_0)^{n-1} = \int_C (z-2-i)^{n-1}$.