

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
2018 Fall MATH2230
Homework Set 2 (Due on Sept. 24)

All the homework problems are taken from Complex Variables and Applications, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P30-31

1. Find the square roots of

(a) $2i$; (b) $1 - \sqrt{3}i$;

and express them in rectangular coordinates.

3. Find $(-8 - 8\sqrt{3}i)^{1/4}$ express the roots in rectangular coordinates, exhibit them as the vertices of a certain square and point out which is the principal root.

P35

4(c). Sketch the closure of the set $\operatorname{Re}\left(\frac{1}{z}\right) \leq \frac{1}{2}$.

P89

1. Show that

(a) $\exp(2 \pm 3\pi i) = -e^2$; (b) $\exp\left(\frac{2 + \pi i}{4}\right) = \sqrt{\frac{e}{2}}(1 + i)$; (c) $\exp(z + \pi i) = -\exp z$.

P109

16. With the aid of expression $\cos z = \cos x \cosh y - i \sin x \sinh y$ ($z = x + yi$) show that the roots of the equation $\cos z = 2$ are

$z = 2n\pi + i \cosh^{-1} 2$ ($n = 0, \pm 1, \pm 2, \dots$).

Then express them in the form $z = 2n\pi \pm i \ln(2 + \sqrt{3})$ ($n = 0, \pm 1, \pm 2, \dots$).

P112

16. By using one of the identities

$$\sinh z = \sinh x \cosh y + i \cosh x \sin y \quad \text{or} \quad \cosh z = \cosh x \cos y + i \sinh x \sin y,$$

find all roots of the equation

(a) $\sinh z = i$; (b) $\cosh z = 1/2$.

17. Find all roots of the equation $\cosh z = -2$.