MMAT5390: Mathematical Image Processing

Special Assignment

Due: 24 March 2021

1. Consider a $N \times N$ image, where $N > 100000$. Suppose the singular value decomposition of I is given by $I = V \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \end{pmatrix} V^T$. It is known that V is given by:

$$V = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 & \cdots & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 & \cdots & 0 \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} & 0 & \cdots & 0 \\ 0 & 0 & 0 & a_{1,1} & \cdots & a_{1,N-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & a_{N-3,1} & \cdots & a_{N-3,N-3} \end{pmatrix}$$

Now, suppose the image I is corrupted by noise $n \in M_{N\times N}(\mathbb{R})$ at the upper left corner. In other words, the noisy image is given by $\tilde{I} = I + n$, where n is given by:

$$n = \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 & 0 & \cdots & 0 \\ \epsilon_2 & \epsilon_1 & \epsilon_3 & 0 & \cdots & 0 \\ \epsilon_3 & \epsilon_3 & \epsilon_3 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Provide that $\epsilon_3 = \frac{\epsilon_1+\epsilon_2}{2}$.

Compute the singular value decomposition of $\tilde{I}$ in terms of $V, \epsilon_1, \epsilon_2, \epsilon_3$ and σ_i's. Please explain your answer with details.