MATH2010 Advanced Calculus I, 2014-15

Homework 07

1. Find the linear approximation $L(x,y)$ of the function $f(x,y)$ at each point

(a) $f(x,y) = e^x \cos y$ at $(0, \pi/2)$
(b) $f(x,y) = x^3 y^4$ at $(1,1)$

2. Similar to the lecture,

$$L(x,y,z) = f(P_0) + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0)$$

is called the linear approximation of $f(x,y,z)$ at $P_0(x_0, y_0, z_0)$. Find the linear approximation $L(x,y,z)$ of the function $f(x,y,z)$ at each point.

(a) $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ at $(1,1,0)$
(b) $f(x,y,z) = e^x + \cos(y+z)$ at $(0, \frac{\pi}{2}, 0)$

3. Let

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2 + y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Show that $f_x(0,0)$ and $f_y(0,0)$ exist, but f is not differentiable at $(0,0)$. (*Hint:* Show that f is not continuous at $(0,0)$.)

4. Let

$$f(x,y) = \begin{cases} 0, & x^2 < y < 2x^2 \\ 1, & \text{otherwise.} \end{cases}$$

Show that $f_x(0,0)$ and $f_y(0,0)$ exist, but f is not differentiable at $(0,0)$.

5. Let

$$f(x,y) = \begin{cases} y^2 + x^2 \sin \dfrac{1}{x}, & x \neq 0 \\ y^2, & x = 0. \end{cases}$$

Show that f is differentiable at $(0,0)$, but is not continuously differentiable in any neighbourhood of $(0,0)$ by proving that $f_x(x,y)$ is not continuous at $(0,0)$.