MATH 2040 Linear Algebra II
2015-16 Term 2
Review Exercise for Midterm 2

All the fields in the questions are either $\mathbb{R}$ or $\mathbb{C}$.

Sec 6.1: # 12, 23.
Sec 6.2: # 2, 3, 17, 22.
Sec 6.3: # 9, 19.
Sec 6.4: # 4, 14, 15.
Sec 6.5: # 2, 3, 11.
Sec 6.6: # 2.

All other questions from practice problem sets 5-9.

Some more computational exercises:

1. Verify each of the following matrices is normal. Find an unitary matrix Q such that Q^*AQ is diagonal, or show that no such Q exists. Is it possible to find such a Q whose entries are real?

   (a) $\begin{pmatrix} 4i & -2 \\ 2 & i \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

   (c) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

   (d) $\begin{pmatrix} 25-16i & 0 & 12i \\ 0 & 25+25i & 0 \\ 12i & 0 & 25-9i \end{pmatrix}$

2. Find a normal matrix with characteristic polynomial t^2+4 and eigenspace $E_{2i} = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 3i \end{pmatrix} \right\}$.

3. Which of the following matrices have an orthonormal eigenbasis over $\mathbb{R}$? Find such an basis if it exists. It is known that all these matrices have characteristic polynomial $p(t) = -t^3 + 3t^2 + 9t - 27$.

$$\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$