

Analysis Seminars on PDEs

Below is a partial list of Analysis Seminars on PDEs organised/coorganised by Renjun Duan. For more information, or if you would like to give a talk or lecture series in CUHK, please freely send an email to rjduan@math.cuhk.edu.hk. You are most welcome to visit us!

Kinetic Lecture Series

March 1, 8 and 15, 2021 (online):

Stability of the shear flows and plasma dynamics in a uniform magnetic field.

Fei Wang

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Abstract: In the first lecture, I will talk about the stability of the shear flows in 2D. To be more specific, we consider Navier-Stokes equation on $\mathbb{T} \times \mathbb{R}$, with initial datum that is epsilon-close to a shear flow $(U(y), 0)$, where $U(y)$ is close to the Couette flow. We prove that if $\epsilon \ll \nu^{1/2}$, where ν denotes the viscosity, then the solution of the Navier-Stokes equation remains epsilon-close to the heat evolution of $(U(y), 0)$ for all $t > 0$, i.e., the stability threshold in finite regularity scales no worse than $\nu^{1/2}$.

In the next two lecture, I will focus on the plasma dynamics. We study the linearized Vlasov equations and the linearized Vlasov-Fokker-Planck equations in the weakly collisional limit in a uniform magnetic field. In both cases, we consider periodic confinement and Maxwellian (or close to Maxwellian) backgrounds. In the collisionless case, for modes transverse to the magnetic field, we provide a precise decomposition into a countably infinite family of standing waves for each spatial mode. These are known as Bernstein modes in the physics literature, though the decomposition is not an obvious consequence of any existing arguments that we are aware of. We show that other modes undergo Landau damping. In the presence of collisions with collision frequency much smaller than 1, we show that these modes undergo uniform Landau damping and enhanced collisional relaxation.

November 15, 2018 (10:30-11:30, AB1 G03):

About the modeling and mathematical analysis of the Boltzmann equation for realistic gases

Laurent Desvillettes

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Abstract: When computing the flow around a shuttle in the upper atmosphere, one has to take into account the fact that the rarefied gas under study contains polyatomic molecules as well as monoatomic molecules. Moreover reactive collisions can have a significant effect on the computation, and cannot be neglected. Writing a coherent set of Boltzmann equations reproducing the complex collisions between the different (possibly reactive) species, raises many modeling issues. The mathematical questions in such a complex situation are also quite interesting, and cannot be answered by a direct extension of techniques coming out of the theory of the Boltzmann equation for a single monoatomic species.

November 12, 2018 (16:30-17:30, LSB 219):

Swarming models with local alignment effects: phase transitions & hydrodynamics

José Antonio Carrillo
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Abstract: We will discuss a collective behavior model in which individuals try to imitate each others' velocity and have a preferred asymptotic speed. It is a variant of the well-known Cucker-Smale model in which the alignment term is localized. We showed that a phase change phenomenon takes place as diffusion decreases, bringing the system from a “disordered” to an “ordered” state. This effect is related to recently noticed phenomena for the diffusive Vicsek model. We analysed the expansion of the large friction limit around the limiting Vicsek model on the sphere leading to the so-called Self-Organized Hydrodynamics (SOH). This talk is based on papers in collaboration with Bostan, and with Barbaro, Caizo and Degond.

November 8, 2018 (11:00-12:00, LSB 219):

Convergence to equilibrium for the solution of the full compressible Navier-Stokes equations

Ruizhao Zi
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Abstract: We study the convergence to equilibrium for the full compressible Navier-Stokes equations on the torus T^3 . Under the conditions that both the density ρ and the temperature θ possess uniform in time positive lower and upper bounds, it is shown that global regular solutions converge to equilibrium with exponential rate. We improve the previous result obtained by Villani in [Mem. Amer. Math. Soc., 202(2009), no. 950] on two levels: weaker conditions on solutions and faster decay rates. This is a joint work with Prof. Zhifei Zhang.

August 25, 2018 (16:00-17:00, LSB 222):

Large-time behavior in hypocoercive BGK-models

Anton Arnold
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Abstract: We shall first discuss hypocoercivity in ODE systems, with the hypocoercivity index characterizing its structural complexity.

BGK equations are kinetic transport equations with a relaxation operator that drives the phase space distribution towards the spatially local equilibrium, a Gaussian with the same macroscopic parameters. Due to the absence of dissipation w.r.t. the spatial direction, convergence to the global equilibrium is only possible thanks to the transport term that mixes various positions. Hence, such models are hypocoercive.

We shall prove exponential convergence towards the equilibrium with explicit rates for several linear, space periodic BGK-models in dimension 1 and 2. Their BGK-operators differ by the number of conserved macroscopic quantities (like mass, momentum, energy), and hence their hypocoercivity

index. Our discussion includes also discrete velocity models, and the local exponential stability of a nonlinear BGK-model.

The proof is based, first, on a Fourier decomposition in space and Hermite function decomposition in velocity. Then, the crucial step is to construct a problem adapted Lyapunov functional, by introducing equivalent norms for each mode.

References:

- F. Achleitner, A. Arnold, E.A. Carlen: On linear hypocoercive BGK models; in Springer Proceedings in Mathematics & Statistics, Vol. 126, 2016; p. 1-37.
- F. Achleitner, A. Arnold, E.A. Carlen: On multi-dimensional hypocoercive BGK models; KRM 11, 2018; p. 953-1009.
- F. Achleitner, A. Arnold, B. Signorello: On optimal decay estimates for ODEs and PDEs with modal decomposition; to appear 2018.

July, 2018 (Half Day on Kinetic Theories SEMINAR in CityU):

Talk 1:

Chemical networks: The case of reaction-diffusion systems

Laurent Desvillettes
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Abstract: Reaction-diffusion systems are naturally linked with networks of chemical reactions. We explain in this talk how the methods based on the entropy enable to obtain explicit quantitative estimates of convergence towards the equilibrium in those systems. The talk is based on results obtained in collaboration with Bao Quoc Tang and Klemens Fellner.

Talk 2:

Entropy dissipation estimates for the relativistic Landau equation and applications

Robert Strain
 University of Pennsylvania, USA
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Abstract: In this talk we will explain very recent results on the relativistic Landau equation (with no spatial dependence). Despite its physical importance, this equation has not received a lot of mathematical attention we think due to the extreme complexity of the relativistic structure of the kernel of the collision operator. In this talk we first largely decompose the structure of the relativistic Landau collision operator. After that we prove the global Entropy dissipation estimate. Then we prove the propagation of any polynomial moment for a weak solution. Lastly we prove the existence of a true weak solution for a large class of initial data. This is a joint work with Maja Taskovic. Our aim is to develop a theory for this understudied and physically important model.

April, 2018

Ivan Moyano
 University of Cambridge
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Part 1 (April 9, 2018, 15:00-17:00, LSB 219):

Classical diffusive limits for the linear Boltzmann equation and study of the Rosseland Approximation in Radiative transfer I

Abstract: We will review some classical and rigorous diffusion limits for the linear Boltzmann equation as studied in a series of papers by Bardos et al. (1985, 2013), both by using Hilbert series expansions and variational (energy) methods. We shall next introduce an example of nonlinear regime (the so-called Radiative Transfer equation) for which the methods mentioned above does not allow to obtain a rigorous diffusive limit (Rosseland approximation).

Part 2 (April 10, 2018, 15:00-17:00, LSB 219):

Classical diffusive limits for the linear Boltzmann equation and study of the Rosseland Approximation in Radiative transfer II

Abstract: In this second lecture, we shall resume our analysis of the Radiative Transfer equation and its diffusive limits following the work by Bardos, Golse, Perthame and Sentis (1988), which crucially relies on averaging lemmas (cf. Golse et al. 1988) to overcome the difficulties encountered concerning the nonlinear regime in the first lecture. If time allows, we shall study how the diffusive limit in the nonlinear regime (Rosseland approximation) may need the introduction of a suitably chosen boundary layer.

February, 2018 (Mini-course):

Stability on the basic waves for the compressible Navier-Stokes equations

Yong Wang

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Abstract: It is well known that we have three kind of basic waves for the Euler system, i.e., shock wave, rarefaction wave and contact discontinuity. In fact, these basic waves capture the large time behavior of the compressible Navier-Stokes equations. In the series lectures, I will show you some stability result on the basic waves for the compressible Navier-Stokes equations. The stability for both the viscous shock wave and contact discontinuity will be reported.

- Lecture 1: February 5, 2018 (Monday) 3:00pm – 5:30pm Room 219
- Lecture 2: February 6, 2018 (Tuesday) 3:00pm – 5:30pm Room 222
- Lecture 3: February 8, 2018 (Thursday) 9:30am – 12:00noon Room 222
- Lecture 4: February 9, 2018 (Friday) 9:30am – 12:00noon Room 219

December 14, 2017 (16:00-17:00, LSB 222):

Pointwise estimate for compressible Navier-Stokes

Shih-Hsien Yu

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Abstract: In this talk we will demonstrate the construction of a Green's function of a compressible Navier-Stokes equation and its application to construct the solution for piecewise smooth initial data.

October 23, 2017 (10:30-11:30, LSB 222):

Consistent BGK models for gas mixtures

Alexander Bobylev

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August 28, 2017 (10:30-11:30, LSB 222):

Solution to the Boltzmann equation without angular cutoff in a critical Chemin-Lerner space

Shota Sakamoto

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Abstract: We consider the Boltzmann equation without angular cutoff near the equilibrium. In this decade, many methods which enable us to extensively analyse the non-cutoff Boltzmann equation have been developed, and among the results, a solution to the equation in a Sobolev space was established. Our aim in this talk is to improve this result from a viewpoint of differentiability of initial datum. For this aim, a Besov space is now known as a strong tool to achieve this, not only for a Cauchy problem of the Boltzmann equation, but also for those of many other PDEs. We construct a solution in a certain time-space-velocity Besov space, which is called a Chemin-Lerner space, and can be thought “critical” for our problem in a sense of inclusion of solution spaces. Proof is based on local existence and continuation argument by a priori estimates, and the latter one will be focused on so that we can see what properties of the Besov space effectively work. This work is based on a joint work with Professor Yoshinori Morimoto at Kyoto University.

August, 2017 (Lecture Series)

Asymptotic methods in kinetic theory of gases: An introduction

Kazuo Aoki

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Abstract:

Part 1 (14 August 2017, 14:30-16:00 LSB 219): The first half of this part is devoted to a brief introduction to kinetic theory of gases, which contains a summary of the Boltzmann equation and its basic properties, the boundary conditions, etc. Then, we consider the free-molecular gas (or the Knudsen gas), i.e., a gas which is so rarefied that the collisions between gas molecules can be neglected (that is, the mean free path of the gas molecules is infinitely long compared with the characteristic length of the system). We present an exact solution that describes the effect of boundary temperature in a quite general situation.

Part 2 (15 August, 2017, 14:30-16:00 LSB 219): In this part, we consider the near continuum regime (or near the fluid-dynamic limit), i.e., the case where the mean free path is small compared with the characteristic length. We show the outline of the formal asymptotic analysis of the steady boundary-value problem of the Boltzmann equation that provides the fluid-dynamic type equations, their boundary conditions of slip type, and the kinetic correction to fluid-dynamic solutions in the vicinity of the boundary (the Knudsen layer) systematically. A special emphasis is put on the case

in which the fluid-dynamic limit thus obtained is not covered by the conventional fluid dynamics (the ghost effect).

April 26, 2017 (15:30-16:30, AB1 501A):

New estimates for the Landau equation of kinetic theory

Laurent Desvillettes

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Abstract: The Landau equation (with Coulomb potential) is a model describing the collisions between charged particles in a plasma. We present new estimates enabling to bound below the entropy dissipation appearing in Landau equation in terms of a weighted H^1 norm (this is a variant of the so-called Cercignani's conjecture). As a consequence, we get new results of smoothness for the solution of the (spatially homogeneous) Landau equation, and of large time behavior for the same equation.

March-April, 2017 (Lecture Series)

Scaling Limits and Effective Equations in Kinetic Theory

Mario Pulvirenti

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Abstract: Classical particle systems and statistical description. Liouville equation and hierarchy. The paradigm of Kinetic Theory and the concept of propagation of chaos.

1. Mean-Field limit and derivation of the Vlasov equation.
2. Boltzmann equation: heuristics. Hard-sphere dynamics. Hierarchies. Derivation of the Boltzmann equation: the Lanford's result. Tree expansion and quantitative estimates.
3. Weak coupling limit. The Landau equation. Heuristics. Grazing collision limit.

Lecture 1: 29 March 2017, 14:30-17:00, LSB 219

Lecture 2: 31 March 2017, 14:30-17:00, LSB 222

Lecture 3: 3 April 2017, 14:30-17:00, LSB 222

Lecture 4: 5 April 2017, 14:30-17:00, LSB 219

March 1, 2017 (Lecture Series: Part I: 10:00-12:00, LSB 219; Part II: 15:00-17:00, LSB 219):

Time periodic problem of the compressible Navier-Stokes equation on the whole space

Kazuyuki Tsuda

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Abstract: The time-periodic problem on the compressible Navier-Stokes equations on an unbounded domain is considered. As the first step of the analysis the whole space is treated. In this case Ma, Ukai and Yang (2010) showed that for spatial dimensions greater than or equal to 5 the existence and stability of time-periodic solutions can be obtained under small time-periodic external

forces. Hence it has remained unknown to solve the time-periodic problem in lower dimensional case. In these lectures I will talk about the existence and stability of time-periodic solutions when the dimensions are greater than or equal to 3.

January, 2017 (Mini-Course)

Some energy methods for viscous conservation laws

Akitaka Matsumura
 Professor Emeritus of Osaka University
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Abstract: Through some typical Cauchy problems in the field of viscous conservation laws, this short course aims to introduce several elementary energy methods which provide desired “a priori” estimates for the global existence in time and asymptotic behavior of solutions.

- Lecture 1: January 09, 2017, 14:30-17:00, LSB 222
- Lecture 2: January 10, 2017, 14:30-17:00, LSB 222
- Lecture 3: January 11, 2017, 14:30-17:00, LSB 219
- Lecture 4: January 12, 2017, 14:30-17:00, LSB 219
- Lecture 5: January 13, 2017, 14:30-17:00, LSB 219

January 13, 2017 (14:00-14:50, LSB 222):

Isentropic compressible Euler system with source terms

Feimin Huang
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Abstract: In this talk, we develop a new technique to prove the global existence of entropy solutions to an inhomogeneous isentropic compressible Euler equations through the compensated compactness and vanishing viscosity method.

January 12, 2017 (10:30-11:30, LSB 219):

Boltzmann collision operator for the infinite range potential

Jin-Cheng Jiang
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Abstract: The conventional Boltzmann collision operator for the infinite range inverse power law model was derived by Maxwell by adopting a collision kernel which is a limit of that for the finite range model by ignoring the glancing angles. Since the interpretation of collision operator for the infinite range potential through limit process to the one with finite range potential is natural in regard to the derivation of the Boltzmann equation. It is the purpose of this work to clarify the physical meaning of the conventional collision operator through the study of the limiting process of the collision operator as the cutoff radius tends to infinity. This is a joint work with Tai-Ping Liu.

January 9, 2017 (14:00-14:50, LSB 222):

Some recent developments of fluids interface problems

Tao Luo

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Abstract: In this talk, I will survey some recent developments of fluids interface problems.

January 12, 2017 (9:00-9:50, LSB 219):

System of conservation laws arising from chemotaxis model

Zhian Wang

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Abstract: We shall present a system of conservation laws derived from the chemotaxis models describing the initiation of tumor angiogenesis. The system has some exclusive features distinguished itself from the existing systems of conservation, which makes the analysis in multi-dimensional spaces very challenging. In the talks, we shall discuss the system background, the development of mathematical results, interesting open questions and challenges faced.

January 11, 2017 (14:00-14:50, LSB 219):

Local-in-time well-posedness theory for MHD boundary layer in Sobolev spaces

Feng Xie

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Abstract: We study the well-posedness theory for MHD boundary layer. Such a kind of boundary layer equations are governed by the Prandtl type equations, which are derived from MHD with no-slip boundary condition for velocity and perfectly conducting conditions for magnetic fields. Under the assumptions that the initial tangential magnetic is always non-zero, we can establish the local-in-time existence, uniqueness, and stability of solutions for the nonlinear MHD boundary layer equations in weighted Sobolev spaces. This talk is based on the joint work with Dr. Chengjie Liu and Professor Tong Yang from CityU of Hong Kong.

January 12, 2017 (14:00-14:50, LSB 222):

Measure valued solutions to the Boltzmann equation

Tong Yang

City University of Hong Kong

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Abstract: In this talk, we will present some recent results on the well-posedness, regularity and large time behavior of the measure valued solutions. The results are from several joint works with Yong-Kum Cho, Yoshinori Morimoto, Shuaikun Wang and Huijiang Zhao.

December 7, 2016 (10:30-11:30, AB1 501a):

Gevrey regularizing effect for the Landau equation with Maxwellian molecules

Chaojiang Xu
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Abstract: In this work, for nonlinear Landau equation with Maxwellian molecules, we construct a global solution in Gevrey class with an initial data in Sobolev space. It is different then other work about regularization of weak or finite order smooth solution. By using the uniqueness of solution, we have proven the Gevrey regularizing effect for the Landau equation.

December 6, 2016 (10:30-11:30, LSB 219):

Entropy-structure preserving numerical schemes for nonlinear diffusive equations

Ansgar Jüngel
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Abstract: Bakry and Emery developed a very efficient tool for the large-time asymptotics of nonlinear diffusion equations and, simultaneously, to derive convex Sobolev inequalities. This method is based on estimating the convexity of the so-called entropy functional. For numerical purposes, it is desirable to devise numerical approximations which preserve this entropy structure. In this talk, some results towards discrete entropy methods are detailed.

Numerical methods include implicit Runge-Kutta and one-leg multi-step time approximations and finite-difference space discretizations. The proofs are based on Dahlquist's G-stability theory, systematic integration by parts, and discrete gradient-flow structures. Examples include the porous-medium equations, cross-diffusion population systems, and nonlinear Fokker-Planck equations.

October 26, 2016 (15:30-16:30, LSB 219):

Qualitative Study of 2D Schrödinger Equation with Exponential Nonlinearity

Hajer Bahouri
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Abstract: The aim of this lecture is to investigate the behavior of 2D Schrödinger equation with exponential nonlinearity. Such equations arise in 3D nonlinear optics problems. The analysis we conducted in this work emphasizes that the nonlinear effect in this framework only stems from the 1-oscillating component of the sequence of the Cauchy data, namely the component which is asymptotically localized in frequency space in a unit ring.

August 2016 (Lecture Series):

Some problems on the neutron transport equation

Xiongfeng Yang
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Abstract: In the talks, some new results about the neutron transport equation will be presented. I focus on the geometric correction for the diffusion limit of the neutron transport equation in disk and in annuli. The BV-regularity of the solution in non-convex domain will be also showed. It will be divided into the following parts.

1. Introduction of the neutron transport equation and the existence of the solutions;
2. The analysis of the asymptotic behavior of the solution and the Milne problem about the boundary layer problem;
3. The estimates of the remained part and the completeness of the diffusion limit for the neutron transport equation;
4. The BV-regularity of the solution for the neutron transport equation in non-convex domain with diffusive boundary condition.

Lecture 1: 17 August 2016, 15:00-17:00, LSB 222

Lecture 2: 19 August 2016, 15:00-17:00, LSB 222

Lecture 3: 24 August 2016, 15:00-17:00, LSB 222

Lecture 4: 26 August 2016, 15:00-17:00, LSB 222

September 7, 2016 (9:20-10:10, AB1 501A):

Regularity of stationary solutions to the linearized Boltzmann equations

I-Kun Chen
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Abstract: We consider the regularity of solutions to the stationary linearized Boltzmann equations in bounded C^1 convex domains in \mathbb{R}^3 for gases with cutoff hard potential and cutoff Maxwellian gases. Suppose that a solution has a bounded weighted L^2 norm in space and velocity with the weight of collision frequency, which is a typical functional space for existence results for boundary value problems. We prove that this solution is Hölder continuous with order $\frac{1}{2}^-$ away from the boundary provided the incoming data have the same regularity and uniformly bounded by a fixed function in velocity with finite weighted L^2 norm with the weight of collision frequency. A smoothing effect due to the combination of collision and transport is used in the proof.

September 7, 2016 (10:25-11:15, AB1 501A):

The initial boundary value problem for the Boltzmann equation with soft potential

Shuangqian Liu
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Abstract: Boundary effects are central to the dynamics of the dilute particles governed by Boltzmann equation. In this talk, we will report some recent studies on both the diffuse reflection and the specular reflection boundary value problems for Boltzmann equation with soft potential. For

the diffuse reflection boundary condition, we capture some new properties of the probability integrals along the stochastic cycles and improve the $L^2 - L^\infty$ theory to give a more direct approach to obtain the global existence and time decay rate. As to the specular reflection condition, our key contribution is to develop a new time-velocity weighted L^∞ theory so that we could deal with the greater difficulties stemmed from the complicated velocity relations among the specular cycles and the zero lower bound of the collision frequency. These methods in the latter case can be applied to Boltzmann equation with soft potential for all other types of boundary condition.

September 7, 2016 (11:15-12:05, AB1 501A):

Probability measures with finite moments and the homogeneous Boltzmann equation

Shuaikun Wang

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Abstract: We characterize the class of probability measures possessing finite moments of an arbitrary positive order in terms of the symmetric difference operators of their Fourier transforms. As an application, we prove the continuity of probability densities associated with measure-valued solutions to the Cauchy problem for the homogeneous Boltzmann equation with Maxwellian molecules. This is a joint work with Prof. Yong-Kum Cho, Prof. Yosinori Morimoto, and Prof. Tong Yang.

September 7, 2016 (14:00-14:50, AB1 501A):

Global well-posedness and regularity of the Boltzmann equation with large amplitude initial data

Yong Wang

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Abstract: The global well-posedness of the Boltzmann equation with initial data of large amplitude has remained a long-standing open problem. In this paper, by developing a new $L_x^\infty L_v^1 \cap L_{x,v}^\infty$ approach, we prove the global existence and uniqueness of mild solutions to the Boltzmann equation in the whole space or torus for a class of initial data with bounded velocity-weighted L^∞ norm under some smallness condition on $L_x^1 L_v^\infty$ norm as well as defect mass, energy and entropy so that the initial data allow large amplitude oscillations. Both the hard and soft potentials with angular cut-off are considered, and the large time behavior of solutions in $L_{x,v}^\infty$ norm with explicit rates of convergence is also studied.

September 7, 2016 (14:50-17:40, AB1 501A):

Nonlinear stability of the 1D Boltzmann equation in a periodic box

Kung-Chien Wu

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Abstract: We study the nonlinear stability of the Boltzmann equation in the 1D periodic box with size $1/\epsilon$, where $0 < \epsilon \ll 1$ is the Knudsen number. The convergence rate is $(1+t)^{-1/2} \ln(1+t)$ for

small time region and exponential for large time region. Moreover, the exponential rate depends on the size of the domain (Knudsen number). This problem is highly nonlinear and hence we need more careful analysis to control the nonlinear term.

September 7, 2016 (16:10-17:00, AB1 501A):

Spectrum analysis of some kinetic equations

Hongjun Yu
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Abstract: In this talk we analyze the spectrum structure of some kinetic equations qualitatively by using semigroup theory and linear operator perturbation theory. The models we consider include the classical Boltzmann equation for hard potentials with or without angular cutoff and the Landau equation with appropriate potentials.

July 20, 2016 (10:30-11:30, AB1 501a):

Sharp bounds for Boltzmann and Landau collision operators

Lingbing He
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Abstract: In this talk, we will provide a stable method to get sharp bounds for Boltzmann and Landau operators in weighted Sobolev spaces and in anisotropic spaces. The main ingredients are two types of dyadic decompositions performed in both phase and frequency spaces and also the geometric decomposition to catch the main structure of the operator. As applications, we will show that our results are related closely to the asymptotic problem of grazing collisions limit and the asymptotics of the Boltzmann equation from short-range interactions to long-range interactions.

March 2, 2016 (09:50-12:00, Room 515 Lee Sau Kee Building):

Session I (09:50-10:50):

Spectrum Structure and Behavior of the Vlasov-Maxwell-Boltzmann System without Angular Cutoff

Yongting Huang
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Abstract: The spectrum structure and behavior of the Vlasov-Maxwell-Boltzmann (VMB) system with physical angular non-cutoff intermolecular collisions are studied in this paper. The analysis shows the effect of the Lorentz force induced by the electro-magnetic field leads to some different spectrum structure from the non-cutoff Boltzmann equation. The spectrum structure in high frequency, quite different from the VMB system with angular cutoff assumption, also illustrates the hyperbolic structure of the Maxwell equation. Furthermore, the large time behaviors and the optimal convergence rates to the equilibrium of the non-cutoff VMB system are established on the spectrum analysis.

Session II (11:00-12:00):

Measure valued solutions to spatially homogenous Boltzmann equation

Shuaikun Wang
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Abstract: In this talk, we study the solutions, which are probability measures with some finite moments, to the non-cutoff homogenous Boltzmann equation. In the Maxwellian molecules case and the moderately soft potentials, the infinite energy solutions are not excluded. Moreover, the smooth effect of the solution is discussed if the initial datum is not a single Dirac mass.

February 24, 2016 (14:30-16:00, LHC G06):

Spectrum analysis of some kinetic equations. Part II

Hongjun Yu
 South China Normal University
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Abstract: We will discuss about the spectrum structure of some kinetic equations. The main analysis tool is the semi-group theory and the linear operator perturbation theory.

February 19, 2016 (14:30-16:00, LSB 219):

Spectrum analysis of some kinetic equations. Part I

Hongjun Yu
 South China Normal University
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Abstract: We will discuss about the spectrum structure of some kinetic equations. The main analysis tool is the semi-group theory and the linear operator perturbation theory.

February 3, 2016 (10:45-11:45, Room 501a, AB1):

Entropy, Duality & Cross Diffusion

Ayman Moussa
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Abstract: Reaction Cross Diffusion systems have been introduced in population dynamics by Shigesada et. al. to take into account the influence of population on inter and intra diffusions rates (cross or self diffusion). After a rather general presentation of these system we will focus on a family of systems generalizing the so called SKT model. We will prove existence of global weak solutions, in arbitrary space dimension. This existence proof relies on the following ingredient : a hidden entropy structure of cross diffusion systems, the Duality Lemma of Michel Pierre and a non linear version of the Aubin-Lions Lemma for degenerate parabolic equations. We will also sketch a simple approximation procedure allowing to keep track of all these estimates.

January 27, 2016 (9:30-10:30, LSB 222):

Commutator estimates with fractional derivatives and local /global existence for the MHD equations

Zaihong Jiang
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Abstract: We establish the local/global-in-time existence of solutions to the MHD equations with fractional diffusion. Firstly, the local-in-time existence is given by combining the commutator estimate and energy method. Secondly, we obtain the global existence of the system with some special diffusion term.

January 20, 2016 (11:00-12:00, LSB 222):

The incompressible limit in L^p type critical spaces

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Abstract: This talk aims at justifying the low Mach number convergence to the incompressible Navier-Stokes equations for viscous compressible flows in the *ill-prepared data* case. The fluid domain is either the whole space, or the torus. A number of works have been dedicated to this classical issue, all of them being, to our knowledge, related to L^2 spaces and to energy type arguments. In the present work, we investigate the low Mach number convergence in the L^p type critical regularity framework. More precisely, in the barotropic case, the divergence-free part of the initial velocity field just has to be bounded in the critical Besov space $\dot{B}_{p,r}^{d/p-1} \cap \dot{B}_{\infty,1}^{-1}$ for some suitable $(p,r) \in [2,4] \times [1,+\infty]$. We still require L^2 type bounds on the low frequencies of the potential part of the velocity and on the density, though, an assumption which seems to be unavoidable in the ill-prepared data framework, because of acoustic waves.

January 19, 2016 (10:45-11:45, Room 501a, Academic Building No. 1):

Homogeneous solutions of stationary Navier-Stokes equations with singular rays

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Abstract: For 3-D stationary Navier-Stokes equations, the explicit (-1) homogeneous solutions is obtained by Landau in 1944 under some delicate assumptions. Tian & Xin proved that all the (-1) homogeneous, axisymmetric, smooth except one-point solutions are Landau solutions. Sverak obtained the same result without the assumption of axisymmetry. We analyse the kernel of the linearized problem and construct various (-1) -homogeneous solutions of stationary Navier-Stokes equations with singular rays. For instance, we obtain a three parameter family of solutions of 3-d stationary NSE, which are (-1) -homogeneous, axisymmetric, and with possibly nonzero swirl. This is a joint work with Yanyan Li and Xukai Yan.

January 13, 2016 (10:00-12:00, LSB 222):

Nonlinear stability and convergence rate to traveling waves for generalized Benjamin-Bona-Mahony-Burgers equations

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Abstract: We are concerned with the large time behavior of traveling wave solutions to the Cauchy problem of generalized Benjamin-Bona-Mahony-Burgers equations

$$u_t - u_{txx} - \nu u_{xx} + \beta u_x + f(u)_x = 0, \quad t > 0, x \in \mathbb{R},$$

with prescribed initial data

$$u(x, 0) = u_0(x) \rightarrow u_{\pm}, \quad \text{as } x \rightarrow \pm\infty.$$

Here u_{\pm} are two given constants and the nonlinear function $f(\cdot) \in C^2(\mathbb{R})$ is assumed to be either convex or concave. An algebraic time decay rate to traveling waves of the solutions of the Cauchy problem of generalized Benjamin-Bona-Mahony-Burgers equation is obtained by employing the weighted energy method developed by Kawashima and Matsumura into discuss the asymptotic behavior of traveling wave solutions to the Burgers equation.

November 13, 2015 (10:30-12:00, LSB 222):

3D axisymmetric Navier-Stokes equations

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Abstract: In this talk, we study the three-dimensional axisymmetric Navier-Stokes system with nonzero swirl. By establishing a new key inequality for the pair $(\frac{\omega^r}{r}, \frac{\omega^\theta}{r})$, we get several Prodi-Serrin type regularity criteria based on the angular velocity, u^θ . Moreover, we obtain the global well-posedness result if the initial angular velocity u_0^θ is appropriate small in the critical space $L^3(\mathbb{R}^3)$. Furthermore, we investigate the global well-posedness for the 3-D inhomogeneous incompressible Navier-Stokes system with the axisymmetric initial data. We prove the global well-posedness provided that

$$\|\frac{a_0}{r}\|_\infty \text{ and } \|u_0^\theta\|_3 \text{ are sufficiently small.}$$

If $\mathbf{u}_0 \in L^1$ and $ru_0^\theta \in L^1 \cap L^2$, we have

$$\|u^\theta(t)\|_2^2 + \langle t \rangle \|\nabla(u^\theta \mathbf{e}_\theta)(t)\|_2^2 + t \langle t \rangle (\|u_t^\theta(t)\|_2^2 + \|\Delta(u^\theta \mathbf{e}_\theta)(t)\|_2^2) \leq C \langle t \rangle^{-\frac{5}{2}}, \quad \forall t > 0.$$

November 10, 2015 (10:30-11:30, LSB 222):

Periodic minimizers in atomistic systems

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Abstract: We consider point configurations x_1, \dots, x_n in R^d , $d = 1, 2, 3$. The existence of periodic minimizers for atomistic energies

$$E = \sum_{i,j} v(|x_i - x_j|),$$

where v is a pair potential like the Lennard-Jones potential, is open in most cases. I will discuss recent progress which gives an affirmative answer in situations where v decays with a sufficiently high polynomial degree. Many ideas are based on analytical tools like rigidity theorems.

September 9, 2015 (10:30-11:30, LSB 222):

Discrete Beckner Inequalities via a Bochner-Bakry-Emery Method for Markov Chains

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Abstract: Beckner inequalities, which interpolate between the logarithmic Sobolev Inequalities and Poincare inequalities, are derived in the context of Markov chains. The proof is based on the Bakry-Emery method and the use of discrete Bochner-type inequalities. We apply our result to several Markov chains including Birth-Death process, Zero-Range process, Bernoulli-Laplace models and Random Transposition models and thus get the exponential convergence rates of the "distributions" of these Markov chains to their invariant measures.

September 2, 2015 (10:00-11:30, LSB 222):

Well-posedness and asymptotic analysis of the Boltzmann equation without angular cutoff

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Abstract: We consider the well-posedness and asymptotics of grazing collisions limit of the Boltzmann equation without angular cutoff. Based on the recent progress of the sharp bounds on the collision operators, we solve the Cauchy problem to the Boltzmann equation in the grazing collisions limit and then give the explicit asymptotic formula to describe the limit process from Boltzmann equation to Landau equation.

July 22, 2015 (11:00-12:00, LSB 222):

Pathway-based kinetic-transport model for E.coli chemotaxis

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Abstract: Recently, the biochemical pathways regulating the flagellar motors were uncovered. This knowledge gave rise to a class of kinetic-transport equations, that takes into account an intra-cellular molecular content and which relates the tumbling frequency to this information. It turns out that the tumbling frequency depends on the chemotactic signal, and not on its gradient.

We first derive the standard Kinetic-transport model which heuristically includes tumbling frequencies depending on the path-wise gradient of chemotactic signal, after appropriate rescaling. The main difficulty is to explain why the path-wise gradient of chemotactic signal can arise in this asymptotic process. And then build a new kinetic system of PBMFT under the assumption that the methylation level is locally concentrated, whose turning operator takes into account the dynamical intracellular pathway. We recover the PBMFT proposed by Si et al. as the hyperbolic limit and connect to the Keller-Segel equation as the parabolic limit of this new model. An augmented Keller-Segel equation with macroscopic intercellular signaling pathway dynamics, which can explain the experimental observation of in fast varying environment is proposed.

July 15, 2015 (10:00-12:00, LSB 222):

Pathway-based kinetic-transport model for E.coli chemotaxis

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Abstract: Ito's calculus says that the solution of diffusion equation can be represented by the solution of the Ito's stochastic equation. Especially, the Feynman-Kac formula is the one of them. Here I introduce the diffusion stochastic process and its application to the VFPF equation.

June, 2015 (Lecture Series)

Dispersive Limit of the Euler-Poisson System

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Abstract: For a wide class of nonlinear systems, if the nonlinearity is weak and if one makes the long-wavelength approximation, the governing equations can be reduced to either the Korteweg-de Vries (KdV) equation or the Burgers equation depending on whether the system is dispersion or dissipation dominated. In this lecture, we will discuss such a long wavelength limit formally and rigorously from the Euler-Poisson equation for ions to the dispersive equations like KdV, Kadomtsev-Petviashvili (KP) and the Zakharov-Kuznetsov (ZK).

Part I: 8 June 2015, 10:00-12:00, LSB 219

Part II: 9 June 2015, 14:30-16:30, LSB 219

April 28, 2015 (10:30-11:30, AB1 501a):

Stability condition for a system of damped wave equations with delay in the coupling

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Abstract: In this talk, we discuss a coupled system of linear damped wave equations with time-delay terms. There are a lot of physical models which contains a time-delay property, and it is important to control the delay effect by using the dissipation terms. To this end, we first consider

the corresponding ordinary differential system and derive some stability conditions for coefficients. Moreover, we apply this result to our PDE problem.

March 6, 2015 (16:00-17:00, LSB 222):

Mathematical analysis of a PDE system for biological network formulation

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Abstract: Motivated by recent physics papers describing basic principles for biological network formation, we study an elliptic-parabolic system of partial differential equations proposed by D. Hu and D. Cai in 2012. The model describes the pressure field by a Darcy's type equation and the dynamics of the network conductance under pressure force effects with diffusion representing randomness in the material structure. We prove the existence of global weak solutions and of local mild solutions and study their long time behavior. It turns out that, by energy dissipation, steady states play a central role in understanding the pattern capacity of the system. We show that for a large diffusion coefficient, the zero steady state is stable. Patterns occur for small values of the diffusion coefficient because the zero steady state is Turing unstable in this range; for vanishing diffusion we can exhibit a large class of dynamically stable (in the linearized sense) steady states.

January 14, 2015 (15:00-16:00, AB1 501a):

Some polymeric fluid flow models: steady states & large-time convergence

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Abstract: We consider a dumbbell model for a dilute solution of polymers in a homogeneous fluid. In a micro-macro model, the incompressible Navier-Stokes equation for the fluid flow is coupled to a Fokker-Planck equation for the (microscopic) distribution of the polymeric chains.

First we analyze the linear Fokker-Planck equation for Hookean dumbbells, and in the case of finite extension nonlinear elasticity (FENE) steady states and large-time convergence using entropy methods. In the FENE case the stationary problem is degenerate elliptic, requiring to use weighted Sobolev spaces. In the coupled Hookean case we also show exponential convergence to a homogeneous stationary flow.

References:

A. Arnold, J.A. Carrillo, C. Manzini: Refined long-time asymptotics for some polymeric fluid flow models, *Comm. Math. Sc.* 8, No. 3 (2010) 763-782.

A. Arnold, C. Bardos: Stable steady states of a FENE-dumbbell model for polymeric fluid flows, preprint, 2014.

November 20, 2014 (10:00-12:00, LSB 222):

Asymptotic profiles of solutions to some hyperbolic type equations

Shuichi Kawashima
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September 26, 2014 (15:30-16:30, LSB 219):

Magnetohydrodynamic fluids with zero magnetic diffusivity

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Abstract: Understanding the incompressible/compressible fluid is a fundamental, but challenging, project not only in numerical analysis, but also in theoretical analysis, especially where extra effects, such as the elastic deformation or the magnetic field, interact with the flow. In this talk, the incompressible/compressible magnetohydrodynamic fluid and its associated flow map will be reviewed first. The main object of this talk devotes to understanding incompressible/compressible magnetohydrodynamic fluids with zero magnetic diffusivity (which is equivalent to infinite conductivity).

November, 2013 (Lecture Series)

Shigeru Takata
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Lecture 1 (25 November 2013, 10:00-12:00, AB1501a):

Asymptotic Analyses in Molecular Gas Dynamics: fluid-dynamic limit and some other topics

Abstract: The present talk starts with an introduction to the Molecular Gas Dynamics, which contains

1. Velocity distribution function,
2. Mean free path and the Knudsen number,
3. Boltzmann equation (BE): the governing equation in the gas phase,
4. Boundary condition for BE: the rule of gas-surface interaction,
5. Equilibrium state and fluid-dynamic limit

Then we proceed to the asymptotic theory for slightly rarefied gases, which connects the system of the BE to that of the conventional fluid dynamics. At the last part, as another application of asymptotic analysis, we will discuss the narrow channel approximation to derive the convection-diffusion system from the BE. This part is related to our theoretical works on the thermally driven compressors and gas separator.

Lecture 2 (27 November 2013, 10:00-12:00, AB1501a):

Theory of reciprocity for rarefied gases

Abstract: The present talk focuses on what we call the symmetric relation which gives a theoretical foundation on the reciprocal property of a rarefied gas for the entire range of the Knudsen number in the linearized regime. Starting from a linearized formulation, we derive that relation by a simple argument which makes use of the detailed balance and the self-adjointness of the linearized collision operator. Then, we introduce a set of elementary point sources on the boundary and derive the reciprocal relations as a consequence of the Green's identity. The idea can be applied not only to time-independent but also time-dependent problems. Several application results of the theory will also be provided.

April 24, 2013 (10:30-12:00, AB1 501a):

Boundary Value Problems in Vlasov Theory

Hyung Ju Hwang

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Abstract: We begin with a review on initial-boundary value problems in the Vlasov-Poisson system with boundary conditions. We then investigate the stationary solutions of the Vlasov-Poisson system with diffusive boundary conditions when the distribution function is given as a function of local energy. Existence and uniqueness of solutions are discussed in various settings.