THE CHINESE UNVIERSITY OF HONG KONG
Department of Mathematics

MATH4900H, First term 2020-21

Grades

- 20% Class participation
- 20% Written notes (10% each *2)
- 10% Introductory presentation
- 20% Progress report
- 30% Final presentation

Schedule

Wk	Monday	
1	Sep 7	Organizational meeting + Introductory Lecture
2	Sep 14	Self study, deciding on topics (see Remark 1)
3	Sep 21	Self study
4	Sep 28	Introductory presentation (see Remark 2)
5	Oct 5	Self study, preparation of notes (see Remark 3)
6	Oct 12	Self study, preparation of notes
7	Oct 19	Progress report (see Remark 4)
8	Oct 26 (Holiday)	Self study, preparation of notes
9	Nov 2	Self study, preparation of notes
10	Nov 9	Self study, preparation of notes
11	Nov 16	Self study, preparation of notes
12	Nov 23	Final presentation (see Remark 5)
13	Nov 30	Final presentation II (if necessary)

Remark 1: We discuss topics in knot theory that center around the Alexander polynomial of a knot. We shall look at this invariant from various perspective, and each student will focus on one (or several) of the approaches below.

(1) Alexander polynomial can be defined combinatorically using skein relations. This approach can be generalized to HOMFLY polynomials.

(2) Alexander polynomial can be defined diagrammatically using spanning trees of knot diagrams, resulting in Kauffman's state sum formula.

(3) Alexander polynomial can be defined geometrically using surfaces that bound a given knot. This approach results in many interesting applications that reveals topological information about knots.

(4) Alexander polynomial can be defined algebraically using Fox free calculus.

(5) Alexander polynomial can be defined via covering space techniques in algebraic topology.

(6)* Alexander polynomial can be obtained from the representation of quantum groups

(7)* Alexander polynomial can be viewed as the Euler characteristic of certain homology theory.

* indicates topics that may require a substantial amount of background and efforts.

Group A: (1)+(2)
Group B: (3)
Group C: (4)+(5)

Remark 2: Each student will give a presentation of 15 minutes about what he / she is going to study.

Remark 3: Each student should write up his/her own notes. It should be a summary of the material to be presented, and should be 1 to 2 pages maximum. The notes should be sent to the instructor at least 3 days before the presentations.

Remark 4: Each student will give a presentation of 30 minutes about his / her progress.

Remark 5: Each student will give a presentation of 45 minutes, summarizing what he / she has done.

Remark 6: At least one of the notes should be written in LaTex; at least one of the presentations should be prepared by Beamer.

Remark 7: Listed below are some of the more advanced topics (some of which are research-level projects) that are suitable for students who indicate a mastery of the material in MATH4900.

(1) Crowell-Murasugi's characterization of Alexander polynomial of alternating knots and the trapezoidal conjecture

(2) Crossing number of alternating knots and Tait conjecture

(3) Ozsvath-Szabo's knot/link Floer invariant from a grid diagram

(4) Jones polynomial and Khovanov homology

(5) Higher Alexander invariant

(6) Finite-type invariant

(7) Alexander polynomial of a graph