MATH 2050C Mathematical Analysis I
2018-19 Term 2

Solution to Problem Set 1

Exercise 2

Notation: Ex. = Exercise; A.P. = Algebraic Properties of $\mathbb{R}$.
a) $-(a+b)+(a+b) \overset{\text{A.P.(A4)}}{=\!=\!=} 0$ and $(-a)+(-b)+(a+b) = (-a)+(-b)+a+b = (-a)+a+(-b)+b = 0$. By Exercise 1(a), $-(a+b) = (-a)+(-b)$.
b) $(-a)\cdot(-b) \overset{\text{Ex.1c}}{=\!=\!=} ((-1)\cdot a)\cdot((-1)\cdot b) \overset{\text{Ex.1d}}{=\!=\!=} (-1)\cdot(-1)\cdot a\cdot b = a\cdot b$.
c) If $a \neq 0$, $(1/(-a))\cdot(-a) = 1$ and $-(1/a)\cdot(-a) \overset{\text{Ex.1c}}{=\!=\!=} -(1/a)\cdot(-1)\cdot a \overset{\text{Ex.1d}}{=\!=\!=} (1/a)\cdot a = 1$. By Theorem 2.1.3(a), $1/(-a) = -(1/a)$.
d) $-(a/b)+(a/b) = 0$ and $(-a)/b + a/b \overset{\text{A.P.(D)}}{=\!=\!=} ((-a)+a)\cdot(1/b) = 0$. By Exercise 1(a), $-(a/b) = (-a)/b$.

Exercise 6

Otherwise, suppose that $(p/q)^2 = 6$ so that p, q are integers and no common integer factors other than 1. Since $p^2 = 6q^2$, p is even and q is odd. Denote $p = 2m$ and $q = 2n+1$ for some integers m and n. Since $(2m)^2 = 6\cdot(2n+1)^2$, $6 = 4(m^2 - 6n^2 - 6n)$. This implies that 6 is a multiple of 4. Contradiction.

Exercise 16

a) $x^2 > 3x+4 \Leftrightarrow x^2 - 3x - 4 > 0 \Leftrightarrow (x+1)(x-4) > 0$. By Theorem 2.1.10 there are two cases:

- $x+1 > 0, x-4 > 0 \Rightarrow x > -1, x > 4 \Rightarrow x > 4$;
- $x+1 < 0, x-4 < 0 \Rightarrow x < -1, x < 4 \Rightarrow x < -1$.

The solution set is $(-\infty, -1) \cup (4, +\infty)$.
b) $1 < x^2 < 4 \Leftrightarrow 1 < |x| < 2$. There are two cases:

- $1 < x < 2$;
- $-2 < x < -1$.

The solution set is $(-2,-1)\cup(1,2)$.
c) Observe that 0 is not a solution to the inequality.

- If $x>0$, $1/x<x \Leftrightarrow 1<x^2 \Leftrightarrow 1<|x| \Leftrightarrow 1<x$.
- If $x<0$, $1/x<x \Leftrightarrow 1>x^2 \Leftrightarrow 1>|x| \Leftrightarrow 1>-x \Leftrightarrow x>-1$.

The solution set is $(-1,0)\cup(1,+\infty)$.
d) Observe that 0 is not a solution to the inequality.

- If $x>0$, $1/x<x^2 \Leftrightarrow 1<x^3 \Leftrightarrow (x-1)(x^2+x+1)>0 \Leftrightarrow (x-1)((x+1/2)^2+3/4)>0 \Leftrightarrow x>1$.
- If $x<0$, $1/x<x^2 \Leftrightarrow 1>x^3 \Leftrightarrow (x-1)((x+1/2)^2+3/4)<0 \Leftrightarrow x<1$.

The solution set is $(-\infty,0)\cup(1,+\infty)$.

Exercise 18

Suppose that the conclusion is not true. By property 2.1.5(iii), $a-b>0$. Combining with $a-b\le\varepsilon$ for every $\varepsilon>0$, $a-b$ satisfies the condition of Theorem 2.1.9. Thus $a-b=0$, contradicting with $a>b$. Hence $a\le b$.