

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH 6022B** (Second term, 2021-22)  
**Essay and Presentation**

The course assessment of MATH 6022B will consist of one in-class presentations (which everyone must attend) and a written mathematical essay on selected topics. Note the following important dates:

- *In-class presentation (via ZOOM): April 28, 2022, 3:30PM-6:15PM*
- *Essay submission deadline (by email): May 5, 2022 at 11:59PM*

### **Presentations and Essay**

*Presentations:* Every student should give his/her own presentation in English. The presentations will be evaluated in terms of content, clarity and pedagogy. You should give some motivation and introductory/background material about the chosen topic, highlight some of the most interesting theorems related, and give some connections to other areas if possible. Each presentation should be roughly 45 minutes long.

*Written essay:* You should write up in LaTeX a mathematical article about the your selected topic. The essay should be at least 5 pages long. Each student is responsible for writing up his/her own essay and send it to the instructor via email by the deadline stated above. More specifically, the essay should comprise of precise statements of the problems you have studied, some of the background of the mathematics involved, and at least a sketch of the ideas of the proofs of some major theorems. References should be supplied at the end of the report (which does not count towards the 5-page requirement).

### **Some suggested topics**

You are welcomed to choose a topic of your own to work on. Below are some suggestions:

- Jang's equation plays a major role in the spacetime positive mass theorem and the study of marginally outer trapped surfaces (MOTS). This equation provides an important link between elliptic PDE theory and the geometric properties of MOTS. See, e.g., *Ref: L. Andersson, M. Eichmair, and J. Metzger, "Jang's equation and its applications to marginally outer trapped surfaces", Complex analysis and dynamical systems IV. Part 2, Contemp. Math., vol. 554, Amer. Math. Soc., Providence, RI, 2011, pp. 13–45.*
- The rigidity case of the spacetime positive mass theorem *Ref: R. Beig and P. Chrusciel "Killing vectors in asymptotically flat space-times. I. Asymptotically translational Killing vectors and the rigid positive energy theorem", J. Math. Phys. 37 (1996), no. 4, 1939–1961; L.-H. Huang and D. Lee, "Equality in the spacetime positive mass theorem", Communications in Mathematical Physics 376 (2020), no. 3, 2379–2407.*
- The theory of compactly supported scalar curvature deformations and gluing constructions *J. Corvino, "Scalar curvature deformation and a gluing construction for the Einstein constraint equations", Comm. Math. Phys. 214 (2000), no. 1, 137–189; J. Corvino and R. Schoen, "On the asymptotics for the vacuum Einstein constraint equations", J. Differential Geom. 73 (2006), no. 2, 185–217.*

- The notion of center of mass and its relation to constant mean curvature foliations near infinity  
*Ref: G. Huisken and S.-T. Yau, “Definition of center of mass for isolated physical systems and unique foliations by stable spheres with constant mean curvature”, *Invent. Math.* 124 (1996), no. 1-3, 281–311; L.-H. Huang, “On the center of mass in general relativity”, *Fifth International Congress of Chinese Mathematicians. Part 1, 2*, AMS/IP Stud. Adv. Math., 51, pt. 1, vol. 2, Amer. Math. Soc., Providence, RI, 2012, pp. 575–591.*
- The spinor proof of positive mass theorem *Ref: Chapter 5 of textbook*
- The concept of quasi-local mass *Ref: Chapter 6 of textbook*

## Useful Links

- The Not So Short Introduction to LaTeX (<https://tobi.oetiker.ch/lshort/lshort.pdf>)
- MathSciNet (<https://mathscinet.ams.org/mathscinet/>)