Math4230 Exercise 2

1. (a) Let C be a nonempty, convex set that contains 0 and let $0 \leq \lambda_1 \leq \lambda_2$. Show that $\lambda_1 C \subseteq \lambda_2 C$.

   (b) Let C be a nonempty, convex set. Let $\alpha, \beta \geq 0$. Show that $\alpha C + \beta C \subseteq (\alpha + \beta)C$.

2. Let $\{x_1, ..., x_m\}$ be affinely independent and suppose $x \notin \text{aff}(\{x_1, ..., x_m\})$. Show that $x_1, ..., x_m, x$ are affinely independent.

3. Suppose C is convex with $\dim(C) = m,\ m \geq 1$. Let $\{x_0, x_1, ..., x_m\} \subset C$ be an affinely independent set. Let $\Delta_m := \text{conv}(\{x_0, x_1, ..., x_m\})$. Show that aff$(C)$ =aff(Δ_m) =aff$(\{x_0, x_1, ..., x_m\})$.

4. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function.
   (a) Show that for every $a \in \mathbb{R}$ the *level set* $\{x \in \mathbb{R}^n | f(x) \leq a\}$ is convex.
   (b) Let $C \subseteq \mathbb{R}$ be a convex set. Is it true in general that the inverse image $f^{-1}(C)$ is a convex set?