MATH 4010. Solution 5.

P178. 6 Pf: (Details are omitted)

① $\left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right\rangle$ is an inner product.

② $\left\langle \begin{pmatrix} x \\ 0 \end{pmatrix}, \begin{pmatrix} x \\ 0 \end{pmatrix} \right\rangle = \langle x, x \rangle_X \quad \forall x \in X$

③ $\max\{\|x\|_X, \|y\|_Y\} \le \|x\|_X + \|y\|_Y \le 2\max\{\|x\|_X, \|y\|_Y\}$

④ Using above ineq. to check the Cauchy sequence in $X \times Y$

□

P178. 15 Pf: (a) $f(\lambda) = \|x + \lambda y\|$ is cont. w.r.t. λ and tends to infinity as $\lambda \to \pm\infty$.

(Details are omitted) (triangle ineq.)

(b) Let $\lambda = u + iv$. $\|x+\lambda y\|^2 = u^2 + v^2 + 2\,\mathrm{Re}\langle x, y\rangle u + 2\,\mathrm{Im}\langle x, y\rangle v + \|x\|^2$

$\Rightarrow$ the min satisfies

$u = -\mathrm{Re}\langle x, y\rangle$. $v = -\mathrm{Im}\langle x, y\rangle$.

(c) By (b).

□