

The Chinese University of Hong Kong
Department of Mathematics

MMAT 5140 Probability Theory 2015 - 2016
Suggested Solution to Homework 5

1. P. 182, Q5

(a)

$$E(X) = \sum_{i=1}^N (i) \left(\frac{1}{N}\right) = \frac{N+1}{2}.$$

(b)

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= \sum_{i=1}^N (i^2) \left(\frac{1}{N}\right) - \left(\frac{N+1}{2}\right)^2 \\ &= \frac{1}{6}(N+1)(2N+1) - \left(\frac{N+1}{2}\right)^2 \\ &= \frac{1}{12}(N+1)(N-1). \end{aligned}$$

(c)

$$\sigma_X = \sqrt{\frac{1}{12}(N+1)(N-1)}.$$

2. P. 182, Q7

$$\begin{aligned} \text{Var}(-3X + 5) &= |-3|^2 \text{Var}(X) \\ &= 9E[(X - \mu)^2] \\ &= 9E[X(X-2) + (2-2\mu)X + \mu^2] \\ &= 9(E[X(X-2)] + (2-2\mu)E(X) + \mu^2) \\ &= 9(3 - (2-2)(1) + 1^2) = 36. \end{aligned}$$

3. P. 197, Q7

$$p_X(x) = \begin{cases} C_x^4 (0.6)^x (0.4)^{4-x} & \text{if } x = 0, 1, 2, 3, 4, \\ 0 & \text{otherwise.} \end{cases}$$
$$p_Y(y) = \begin{cases} C_{\frac{y-1}{2}}^4 (0.6)^{\frac{y-1}{2}} (0.4)^{4-\frac{y-1}{2}} & \text{if } y = 1, 3, 5, 7, 9, \\ 0 & \text{otherwise.} \end{cases}$$

4. P. 197, Q10

(a) Probability

$$= \sum_{i=2}^5 C_i^5 \left(\frac{1}{3}\right)^i \left(\frac{2}{3}\right)^{5-i}.$$

(b) Probability

$$= C_2^5 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3.$$