The Chinese University of Hong Kong
Department of Mathematics

MMAT 5140 Probability Theory 2015 - 2016
Suggested Solution to Homework 2

1. P. 34, Q3

   (a) The statement is false. Consider an experiment which picks a person from a group consisting of women only. Clearly, the only possible outcome is that the person picked is a female and let such event be A. Let Ω be the sample space, then $\Omega = \{A\}$ and hence $\Omega \neq A$.

   (b) This statement is also false. Consider picking a real number randomly from 0 to 1, that is, for $0 \leq t \leq t$,
$$P(0 \leq x \leq t) = t.$$
   Let B be the event that $x = 1$, then $P(B) = 0$ but $B \neq \emptyset$.

2. P. 34, Q9 Clearly, $\frac{1}{2} \in \left(\frac{1}{2} - \frac{1}{2n}, \frac{1}{2} + \frac{1}{2n}\right)$ for all $n \in \mathbb{N}$. We need to show that any real number x such that $\left|x - \frac{1}{2}\right| > 0$ is not contained in the intersection, that is, $\frac{1}{2}$ is the only element in the intersection. Let such a real number be given, then there exists an $\varepsilon > 0$ (depending on the given number x) such that $\left|x - \frac{1}{2}\right| > \varepsilon$. Note that for such a *fixed* ε, we can find an N such that $\frac{1}{2N} < \varepsilon$. Hence, $x \notin \left(\frac{1}{2} - \frac{1}{2N}, \frac{1}{2} + \frac{1}{2N}\right)$ and the intersection contains $\frac{1}{2}$ as the only element.

3. P. 122, Q32

   (a)
$$\begin{aligned}
&P(\text{at least one head in the first } n \text{ flips}) \\
&= 1 - P(\text{no head in the first } n \text{ flips}) \\
&= 1 - \left(\frac{1}{2}\right)^n
\end{aligned}$$

   (b) From the definition of Combinations, we have
$$P(\text{exactly } k \text{ heads in the first } n \text{ flips}) = C_k^n \left(\frac{1}{2}\right)^k.$$

(c)

$$\begin{aligned}
&P(\text{getting all heads indefinitely}) \\
&= \lim_{n\to\infty} P(\text{getting all heads in the first } n \text{ flips}) \\
&= \lim_{n\to\infty} \left(\frac{1}{2}\right)^n \\
&= 0
\end{aligned}$$