MMAT5010 2021 Assignment 4

Q1. Recall that

$$||T|| = \sup\{||Tx|| : x \in \mathbb{R}^2,\ ||x|| = 1\}$$

Note if $x = (x_1, x_2) \in \mathbb{R}^2$, then $Tx = (x_1 + 2x_2, 3x_2)$. Thus

$$||Tx|| = \max(|x_1 + 2x_2|,\ |3x_2|) \le \max(|x_1| + 2|x_2|,\ 3|x_2|) \le 3||x||$$

This shows $||T|| \le 3$. We guess that $||T|| = 3$, and to show it, we want to find $x \in \mathbb{R}^2$, $||x|| = 1$ and $||Tx|| = 3$. We may take $x = (0, 1)$ or $(1, 1)$.

Q2. To show T is discontinuous it needs to show that T is unbounded on the ball $B_{c_{00}}$. Let $e_k = (0, 0, \dots, 0, 1, 0, \dots)$ (k-th entry is 1, others are 0). Note that $e_k \in B_{c_{00}}$, and $Te_k = ke_k$, $||Te_k|| = k$. Hence $||T||$ cannot be bounded on $B_{c_{00}}$.