

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH4010 Functional Analysis 2021-22 Term 1**  
**Homework 2**  
**Deadline: 2021-09-27 Monday**

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Show that vectors  $(e_n)$ , where  $e_n$  is the sequence whose  $n$ -th term is 1 and all other terms are zero,

$$\begin{aligned}e_1 &= (1, 0, 0, \dots), \\e_2 &= (0, 1, 0, \dots), \\&\dots\end{aligned}$$

form a Schauder basis in  $\ell^p$  for every  $p \in [1, +\infty)$  and in the spaces  $c_0$  and  $c_{00}$ .

2. Let  $X = \{x \in C[0, 1]: x(0) = 0\}$  with the sup-norm, and let  $f$  be a linear functional on  $X$  defined by

$$f(x) = \int_0^1 x(t) dt.$$

Show that  $\|f\| = 1$ .

— THE END —