

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 (2021-22, Term 2)
Topics in Geometry
Homework 4
Due Date: 21st April 2022

We denote by i the imaginary unit $\sqrt{-1}$, and \mathbb{D} the hyperbolic plane $\{z \in \mathbb{C} : |z| < 1\}$ in the disk model.

1. Find the area of the hyperbolic triangle in \mathbb{D} with vertices at $-1, 1, \frac{i}{\sqrt{3}}$.
2. Find the area of the hyperbolic triangle in \mathbb{D} with vertices at $0, i, 2 - \sqrt{3}$.
3. Show that the sum of exterior angles of a hyperbolic polygon is $2\pi + A$, where A is the hyperbolic area of the polygon.
4. Find the area of a hyperbolic equilateral triangle, each side of which has hyperbolic length $\cosh^{-1}(1 + \sqrt{2})$. (*Hint:* Use the Sine and Cosine Rules.)
5. Let $\triangle ABC$ be a hyperbolic isosceles triangle right angled at C .
 - (a) Suppose the area of $\triangle ABC$ is $\frac{\pi}{6}$. Show that the length of the edge BC (or equivalently AC) is given by $\ln(\sqrt{2} + \sqrt{3})$.
 - (b) Show that the distance from the vertex C to the side AB is less than $\ln(1 + \sqrt{2})$.(*Hint:* Use the Sine and Cosine Rules.)