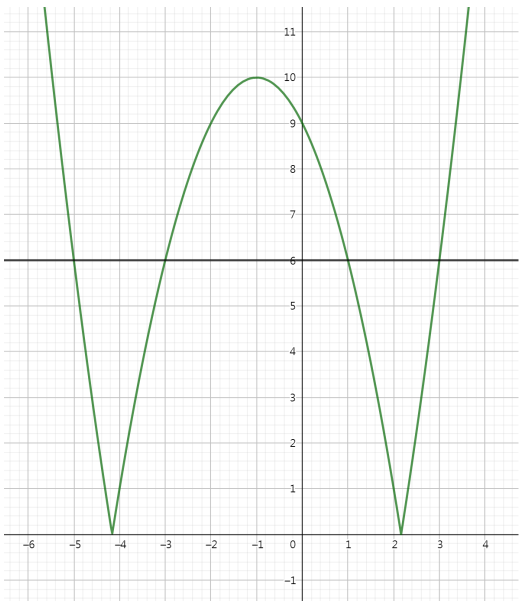
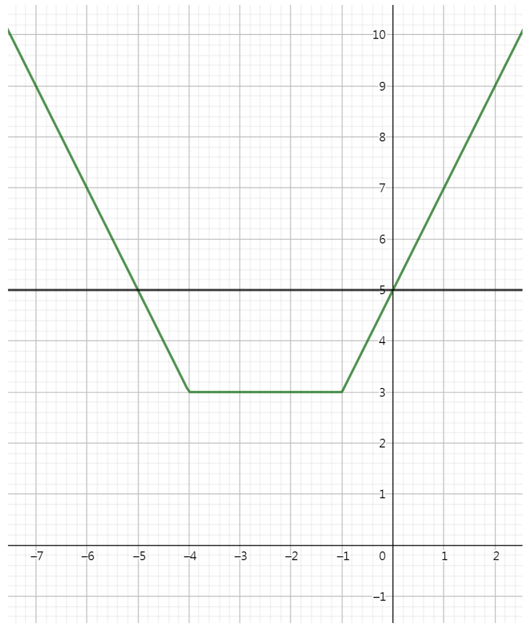
Answers for exercise

1.(a) $x = -5$, $x = -3$, $x = 1$ or $x = 3$



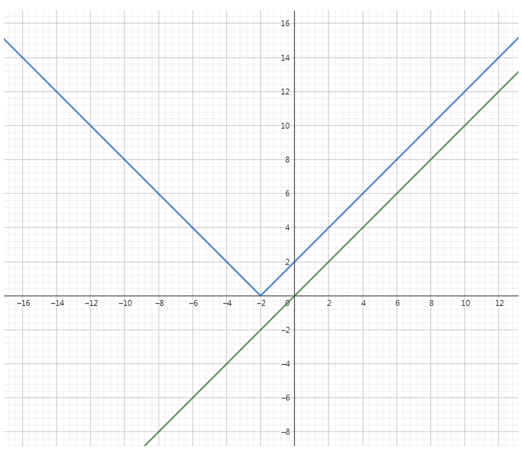
Graph of $y = (x+1)^2 - 10$ and $y = 6$

1.(b) $x = -5$ or $x = 0$



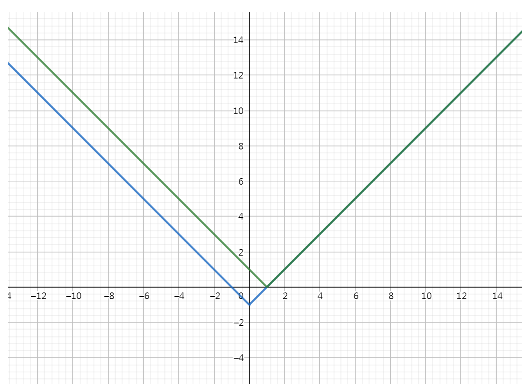
Graph of $y = |x+1| + |x+4|$ and $y = 5$

1.(c) all real values of x



Graph of $y = |x+2|$ and $y = x$

1.(d) all x satisfying $x \geq 1$



Graph of $y = |x-1|$ and $y = |x| - 1$

2.(a) true
2.(b) false
2.(c) false
2.(d) true
2.(e) true
2.(f) true
2.(g) false
2.(h) false
2.(i) true

3.(a) (II)
3.(b) (IV)
3.(c) (I)
3.(d) (III)

4.(a) no real solution
4.(b) $x = -6$ or $x = 2$
4.(c) $x = -3$ or $x = 1$
4.(d) $x = 2$ or $x = 10$
4.(e) $x = -1$ or $x = \frac{5}{2}$
4.(f) $x = -2$ or $x = 2$
4.(g) $x = -1$ or $x = 3$
4.(h) $x = 1$
4.(i) $x = 2$
4.(j) $x = -6$ or $x = -3$ or $x = -1$
4.(k) $x = 0$ or $x = 5$
4.(l) $x = -3$ or $x = 1$
4.(m) $x = -2 - \sqrt{3}$ or $x = 2 - \sqrt{3}$ or $x = -\sqrt{3} - 2$ or $x = \sqrt{3} + 2$
4.(n) $x = -3$ or $x = -2\sqrt{\frac{2}{3}} - 3$ or $x = 2\sqrt{\frac{2}{3}} - 3$
4.(o) $x = 0$ or $x = 1$
4.(p) $x = -\frac{8}{3}$ or $x = 6$
4.(q) $x = -6$ or $x = 4$
4.(r) $x = 0$ or $x = \log 3$
4.(s) $x = -\log 2$ or $x = \log 2$
4.(t) $x = -1$ or $x = \frac{2}{3}$
4.(u) $x = -e^2$ or $x = e^2$
4.(v) $x = \frac{1}{81}$ or $x = 9$
4.(w) $x = -\frac{\pi}{4}$ or $x = \frac{\pi}{4}$
4.(x) $x = -\frac{3\pi}{4}$ or $x = -\frac{\pi}{4}$ or $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

5.(b) We have shown $(|a|)^2 = a^2$.
Also, by 5.(a), $|a| \geq 0$. So $\sqrt{(|a|)^2} = |a|$.
Hence $|a| = \sqrt{(|a|)^2} = \sqrt{a^2}$.

5.(e) If part: Suppose $|a| = 0$.
Assume $a \geq 0$. Then we have $a = 0$.
Assume $a < 0$. Then we have $-a = 0$ and so $a = 0$, which contradicts the assumption.
So we must have $a = 0$.

Only if part: It is obviously that if $a = 0$ then $|a| = 0$.
Hence $|a| = 0$ if and only if $a = 0$.
By replacing a with $a - b$, we obtain $|a - b| = 0$ if and only if $a = b$.

6.(c) Note that if $a, b \geq 0$ or $a, b < 0$, the inequality is true because LHS is equal to RHS.
Now suppose a, b are not of the same sign. Without loss of generality, assume $a \geq 0$ and $b < 0$.
(Case 1) If $a + b \geq 0$, then $|a + b| = a + b$.
By 6.(b), $a \leq |a|$ and $b \leq |b|$. So $|a + b| = a + b \leq |a| + |b|$.
(Case 2) If $a + b < 0$, then $|a + b| = -a - b$.
By 6.(b), $-a \leq |-a| = |a|$ and $-b \leq |-b| = |b|$. So $|a + b| = -a - b \leq |a| + |b|$.
Hence $|a + b| \leq |a| + |b|$ in all cases.

6.(d) From 6.(c), we have $|a| = |(a - b) + b| \leq |a - b| + |b|$.
Then, $|a| - |b| \leq |a - b|$.
From 6.(c), we also have $|b| = |(b - a) + a| \leq |b - a| + |a|$.
Then, $|a| - |b| \geq -|b - a| = -|a - b|$.
Combining the two inequalities, we get $-|a - b| \leq |a| - |b| \leq |a - b|$.
By 6.(a), since $|a - b| \geq 0$, we have $||a| - |b|| \leq |a - b|$ as desired.

7. We only show that $\max(\{a, b\}) = \frac{a+b+|a-b|}{2}$:
(Case 1) Suppose $a \geq b$. Then $|a - b| = a - b$.
So $\frac{a+b+|a-b|}{2} = \frac{a+b+a-b}{2} = a = \max(\{a, b\})$.
(Case 2) Suppose $a < b$. Then $|a - b| = b - a$.
So $\frac{a+b+|a-b|}{2} = \frac{a+b+b-a}{2} = b = \max(\{a, b\})$.
Hence $\max(\{a, b\}) = \frac{a+b+|a-b|}{2}$.

8.(b) Note that $b^2 \geq 0$. So $a^2 \leq a^2 + b^2$.
Also, by 5.(a), $|a| \geq 0$ and $\sqrt{a^2 + b^2} \geq 0$.
Then, $|a| = \sqrt{a^2} \leq \sqrt{a^2 + b^2}$.
Similarly, $|b| \leq \sqrt{a^2 + b^2}$.
Hence by 6.(c), $|a + b| \leq |a| + |b| \leq 2\sqrt{a^2 + b^2}$.

Note that it becomes equality if and only if $a, b = 0$.

8.(c) Since a, b are solutions to $|x + 1| < c$, we have $|a + 1| < c$ and $|b + 1| < c$.
So by 6.(e), $|a - b| \leq |a + 1| + |-1 - b| = |a + 1| + |b + 1| < 2c$.

8.(d) Since $|x - 2| < a$, by 6.(a) we have $-a < x - 2 < a$. So $4 - a < x + 2 < 4 + a$.
Then, by 5.(e), $|x^2 - 4| = |x - 2||x + 2| < a(4 + a) < a^2 + 4a + 4 = (a + 2)^2$.