Binomial theorem exercise

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Quick Review

Binomial Theorem

Let a, b be real numbers, and n be a positive integer. Then

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

or equivalently,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Exercise

1 Expand the following expressions using binomial theorem.

(a) $(a+b)^3$

(b) $(3x-4)^5$

(c) $\left(5x^3-y^2\right)^4$

(d) $\left(2x-\dfrac{1}{x}\right)^4$

2 Expand the following expressions in ascending power of x up to the term x^3.

(a) $(2x+3)(x-3)^4$

(b) $(2-x+x^2)^6$
Hint: You may consider $(2-x+x^2)^6$ as $[2+(-x+x^2)]^6$.

3 Calculate the following values using binomial theorem.

(a) 101^4

(b) 98^3

4 Estimate the following values using binomial theorem.

(a) 1.01^6 up to 3 decimal places.

(b) 1.98^4 up to 2 decimal places.

5 (a) Expand $(1-2x+3x^2)^6$ in ascending power of x up to the term x^3.

(b) Solve the equation $1-2x+3x^2=0.9803$.

(c) By choosing a suitable value in (b) and using part (a), estimate the value of 0.9803^6.

6 Consider the expression $\left(2x^2-\frac{1}{x}\right)^{13}$, find

(a) the coefficient of the term x^{11}.

(b) the coefficient of the term $\frac{1}{x^4}$.

(c) the constant term.

7 Consider the expression $(3-x+2x^2)(5x-1)^8$, find

(a) the coefficient of the term x^{10}.

(b) the coefficient of the term x.

(c) the coefficient of the term x^4.

8 Given that the constant term of the expression $(x^2-2)^n(2x-1)$ is -16, where n is a positive integer.

(a) Find the value of n.

(b) Hence, find the coefficient of the term x^5.

9 Let a, b be integers, and n be a positive integer.

(a) expand $(ax+b)^n$ in ascending power of x up to the term x^3.

(b) Given that the coefficients of the term $\frac{1}{x}$, x, and the constant term of the expression $\left(2x-3+\frac{1}{x}\right)(ax+b)^n$ are -1, -72 and 13 respectively. Find the value of a, b and n.

10 Let n be a positive integer. Explain why

$$\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

11 Let a, b be real numbers,and n be a positive integer. Show that

$$(a+b)^{2n+1} = \sum_{k=0}^{n} \binom{2n+1}{k} (a^k b^{2n+1-k} + a^{2n+1-k} b^k)$$

12 In this questions, we are going to prove the binomial theorem using Mathematical Induction.
Let a, b be real numbers,and n be a positive integer.

(a) Let k and i be positive integers. Show that

$$\binom{k}{i-1} + \binom{k}{i} = \binom{k+1}{i}$$

.

(b) Using the result in part (a), show that for any positive integer n,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

. Hint: You can use the following fact.

$$\sum_{i=0}^{k} \binom{k}{i} a^{i+1} b^{k-i} = \sum_{i=1}^{k+1} \binom{k}{i-1} a^i b^{k+1-i}$$