Analysis of the M and M^2 Routings in Circuit-Switched Networks

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Abstract. In nonhierarchical circuit-switched networks, calls can be routed to alternate paths if the direct path is blocked. In this paper, we analyze two alternate-path routing rules called the *Maximum Free Circuit* routing and the *Maximum Free Circuit with Minimum Occupied channel* routing. For convenience, we shall call them the M and M^2 routings respectively. In the use of M routing, a call is routed to an alternate path that has the maximum number of free circuits when the direct path is blocked. The M^2 routing is an improvement of the M routing in that when multiple alternate paths have the same number of free circuits, the path with the smallest total occupied channels is chosen. Analytical results show that M^2 routing provides additional improvement over M routing when the number of alternate paths is large and/or the trunk group size is small. These results are verified by simulation. As the implementation of M^2 routing is no more complicated than M routing (both require the same channel occupancy information) and its performance is always better than M routing, M^2 routing is deemed a better rule to use.

1. INTRODUCTION

Network management is "the supervision of the telecommunication network to assure the maximum flow of traffic under all conditions" [1]. When an overload occurs, various network management functions must be performed to control the flow of traffic to minimize network congestion. These control functions include the reduction of operator traffic, recorded announcements, alternate route cancellation, traffic rerouting etc. With the use of common channel signaling and stored-program control, more sophisticated control functions can be used in network management. Among these control functions, re-routing of traffic to less congested routes should always be done first, as it affects neither the customers nor the other network management functions.

In recent years, a variety of approaches to alternate routing networks havebeen developed. AT&T has used a decentralized nonhierarchical routing strategy, called *Dynamic Nonhierarchical Routing* (*DNHR*) [2] for a number of years. *DNHR* is a time-dependent routing scheme that increases network efficiency by taking advantage of the noncoincidence of busy hours in a large-toll network. The second approach, which is currently being implemented in the British Telecom main network, is called *Dynamic Alternate Routing* (*DAR*) [3]. The *DAR* scheme has the advantages of (1) distributed control, (2) no need for detailed information passing between nodes and (3) no need for a pre-planning of routing patterns. The *Dynamically Controlled Routing* (*DCR*) [4] proposed by Northern Telecom is a centralized routing rule. A central routing processor receives information every 10 s from all the switches and update their *DCR* tables accordingly. The choice of alternate routes is based on the number of idle trunks and the exchanged utilization levels and is therefore a state-dependent rule. Taking advantage of the fact that it is feasible to monitor channel occupancies and make routing decisions on a call-by-call basis, AT&T recently used a new routing scheme called *Real-Time Network Routing* (*RTNR*) [5] which can improve the network connection availability while simultaneously reducing network costs. In *RTNR*, if a direct path is blocked, the call will be routed to the least congested alternate path.

Previous analytical studies in this area include the work of Krupp [6] on *Random Alternate* routing with and without trunk reservation on symmetrical networks, the extension by Akinpelu [7] on general non-symmetrical networks and the incorporation of external blocking by Yum and Schwartz [8].

In this paper, we analyze the performance of two state-dependent routing procedures on symmetrical fully connected networks. The first one is called *Maximum Free Circuit* routing whose model, as reported in [9], is the first Fixed Point Model for state dependent routing where the rate of the alternate routed traffic offered to an individual link depends on the state of the link. It directs an overflowed call to an alternate path that has the maximum number of free circuits. It was reported in [3] as the *Least Busy Alternate* routing. We choose to call it *Maximum Free Circuit* routing because it is more descriptive. It will also not be confused with the second rule that we are studying in this paper called *Maximum Free Circuit with Minimum Occupied Channel* routing. Here a circuit is defined to be a concatenation of channels on an alternative path. We shall, for convenience, call the first one M routing and the second one M^2 routing. M^2 routing is an improvement of M routing in that when multiple alternate paths have the same number of free circuits, the path with the smallest total occupied channels is chosen. We shall show that the use of these routing procedures together with trunk reservation can indeed give a higher network carrying capacity when compared to the use of direct path routing. M^2 routing as presented in [10] is very similar to *RTNR* [5] and both were presented in the same conference. Due to analytical difficulties, we shall use the same fully-connected, symmetrical, uniformly loaded, nonhierarchical network model used in [6] and [8]. We shall also use the same set of simplifying assumptions in [6-8], namely that the traffic statistics are assumed to be independent at each link and that the alternately routed (or the overflowed) traffic is assumed to be Poisson. A detailed survey on the development of approximate analytical efforts for circuit switched networks can be found in [11].

Recently, Garzia and Lockhart [12] applied Compartmental Modeling to non-hierarchical communications networks. This modeling is much more complicated than ours, but it allows the formulation of network dynamics. Our approach is to derive the steady state performance. More recently, Mitra, Gibbens and Huang [13] proposed a simplified implementation of the M routing based on the aggregation of states. With proper design, it can substantially reduce signaling traffic with only a small loss of performance.

2. M ROUTING

We consider two cases here: without trunk reservation and with trunk reservation.

2.1. *Without trunk reservation*

Consider a E node fully connected and uniformly loaded network where all links consist of N channels. Let P_n be the probability that there are n calls on a link (or that n channels are occupied). Then P_N is the probability of blocking on that link. Let D be the direct-route offered load to a link. Then DP_N is the overflowed load to the alternate paths. We shall restrict our choice of alternate paths consisting of only two links. It was shown [14] that the total number m of such two-link alternate paths is equal to $E-2$.

Consider a particular alternate path. Let the number of occupied channels on the first link be i and that on the second link be j. Then the number of occupied circuits k in that path is $k = \max(i, j)$. When the direct path is full, the M routing will direct the call to the alternate path with the maximum number of free circuits or with minimum k. When there are more than one such paths, choose one at random.

Consider a particular path AC. If link AC is full, the overflowed AC calls of rate DP_N will be routed randomly to one of the *Maximum-Free-Circuit* paths (or M paths for short). Let there be a total of α such M paths. Then, the alternate path load of AC that falls on a particular M path, say path ABC, is DP_N/α. Let Z_k be the probability that a two link alternate path has k or more occupied circuits. Then,

$$Z_k = 1 - \left\{ \text{Prob} \begin{bmatrix} \text{a link has less than } k \\ \text{occupied channels} \end{bmatrix} \right\}^2 = \begin{Bmatrix} 1 & k=0 \\ 1-\left(\sum_{n=0}^{k-1} P_n\right)^2 & 1 \le k \le N \end{Bmatrix} \tag{1}$$

Given that path ABC has k occupied circuits, the probability $f(\alpha|k)$ that the $\alpha - 1$ other alternate paths also have k occupied circuits each *and* each of the remaining $m-\alpha$ alternate paths has more than k occupied circuits is given by

$$f(\alpha \mid k) = \binom{m-1}{\alpha-1} \left(Z_k - Z_{k+1}\right)^{\alpha-1} Z_{k+1}^{m-\alpha} \tag{2}$$

where $Z_k - Z_{k+1}$ is the probability that an alternate path has k occupied circuits. Therefore, given that path ABC has k occupied circuits, the amount of traffic $y(k)$ that gets routed from AC to alternate path ABC is

$$y(k) = \sum_{\alpha=1}^{m} \frac{DP_N}{\alpha} f(\alpha \mid k) = DP_N \frac{Z_k^m - Z_{k+1}^m}{m\left(Z_k - Z_{k+1}\right)} \tag{3}$$

Therefore, given that link AB has i busy channels, the overflowed traffic a_i from link AC to link AB is

$$a_i = \sum_{j=0}^{N-1} y\left(\max(i, j)\right) P_j \qquad 0 \le i \le N-1 \tag{4}$$

Since link AB carries the alternate traffic from 2 m alternate paths, when link AB has i busy channels, the total alternate-route traffic A_i on link AB is

$$A_i = 2\, m\, a_i \tag{5}$$

When links AB has channel occupancy i the call arrival rate λ_i and the call departure rate μ are

$$\begin{aligned} \lambda_i &= D + A_i \qquad & i = 0, 1, \ldots, N-1 \\ \mu_i &= i \qquad & i = 1, 2, \ldots, N \end{aligned} \tag{6}$$

Since the arrival rates are functions of the state probabilities, this "birth-death" process can only be solved numerically by relaxation as follows. From (5), we have

$$\begin{aligned} A_i &= 2\, m \left[\sum_{j=0}^{i} y(j)\, P_j + \sum_{j=i+1}^{N-1} y(j)\, P_j \right] = \\ & 2\, D\, P_N \left(\sum_{j=0}^{i} \frac{Z_i^m - Z_{i+1}^m}{Z_i - Z_{i+1}} P_j + \sum_{j=i+1}^{N-1} \frac{Z_j^m - Z_{j+1}^m}{Z_j - Z_{j+1}} P_j \right) = \\ & 2\, D\, P_N \left[\frac{Z_i^m - Z_{i+1}^m}{Z_i - Z_{i+1}} \left(1 - \sum_{j=i+1}^{N} P_j \right) + \right. \\ & \left. \sum_{j=i+1}^{N-1} \frac{Z_j^m - Z_{j+1}^m}{Z_j - Z_{j+1}} P_j \right] \qquad i = 0, 1, \ldots, N-1 \end{aligned} \tag{7}$$

which, through (1), can be expressed in terms of P_i, P_{i+1}, ... , P_N. Next, the balance equation for the above process says

$$(D + A_i)\, P_i = (i+1)\, P_{i+1} \qquad i = 0, 1, \ldots, N-1 \tag{8}$$

Substituting (7) into (8), we arrive at a set of nonlinear equations. Let $i = N-1$, we obtain a nonlinear equation with two unknowns P_{N-1} and P_N. Assuming an initial value for P_N say equal to $P_N^{(0)}$. Then $P_{N-1}^{(0)}$ can be solved numerically. Repeated use of (8) with $i = N-2$, $i = N-3$,... allows us to solve $P_{N-2}^{(0)}$, $P_{N-3}^{(0)}$, ..., $P_0(0)$. Using the normalization equation $P_N^{(0)}$ can now be updated as

$$P_N^{(1)} = \frac{P_N^{(0)}}{P_N^{(0)} + \sum_{i=0}^{N-1} P_i} \tag{9}$$

Repeat the above iterations until certain accuracy criterion is met for P_N. Following the same argument in [14], the end-to-end blocking probability B_M using M routing is

$$\begin{aligned} B_M &= \text{Prob} \begin{bmatrix} \text{Blocking on} \\ \text{the direct path} \end{bmatrix} \text{Prob} \begin{bmatrix} \text{Blocking on all} \\ m \text{ alternate paths} \end{bmatrix} = \\ & P_N \left[1 - (1 - P_N)^2 \right]^m \end{aligned} \tag{10}$$

For the numerical results presented in section 4, a relative error of less than 10^{-4} was imposed on all end-to-end blocking probabilities.

2.2. *With trunk reservation*

With trunk reservation, the last r free channels on a link are always reserved for direct route traffic. Hence the call arrival and the departure rates on a particular link become

$$\lambda_i = \begin{cases} D + A_i & 0 \le i \le N - r - 1 \\ D & N - r \le i \le N - 1 \end{cases} \tag{11a}$$

$$\mu_i = i \qquad i = 1, 2, \ldots, N \tag{11b}$$

where

$$A_i = \sum_{j=0}^{N-r-1} y\left(\max(i, j)\right) P_j \qquad i = 0, 1, \ldots, N-r-1 \tag{12}$$

For $i > N - r$, we can solve the balance equation directly to obtain P_i in terms of P_{N-r} as follows:

$$P_i = \frac{(N-r)!\, D^{i-N+r}}{i!} P_{N-r} \qquad N - r + 1 \le i \le N \tag{13}$$

Therefore, substituting (13) into (12), A_i can be expressed in terms of P_i, P_{i+1}, ... , P_{N-r}. Substituting A_i into the following balance equations.

$$(D + A_i)\, P_i = (i+1)\, P_{i+1} \qquad i = 0, 1, \ldots, N-r-1 \tag{14}$$

$\{P_i\}$ can similarly be computed as in the last subsection. The end-to-end blocking probability $B_{M/T}$ for M routing with Trunk Reservation is

$$B_{M/T} = P_N \left[1 - \left(1 - \frac{N!\, P_N}{D^N} \sum_{i=N-r}^{N} \frac{D^i}{i!} \right)^2 \right]^m \tag{15}$$

3. M² ROUTING

For M^2 routing, we also derive P_N for the two cases with and without trunk reservation. The end-to-end blocking probabilities, denoted as B_{M^2} and $B_{M^2/T}$, are given by (10) and (15) respectively with the new P_N.

3.1. *Without Trunk Reservation*

Consider one particular alternate path ABC of a direct path AC. Let the number of busy channels on the first and the second links be denoted as i and j respectively. Then $k = \max(i, j)$ and $l = \min(i, j)$ are the occupancies of the more busy and the less busy links respectively.

When the direct path is full, the M^2 routing rule will route the call to the alternate path with minimum k. When there are more than one such path, choose the one with minimum l. When there are more than one path with the same minimum k and minimum l, choose one at random.

Let $Y_{k,l}$ be the probability that path ABC has k and l busy channels on its two links. Then,

$$Y_{k,l} = \begin{Bmatrix} P_k^2 & 1 = l \\ 2\,P_k\,P_l & k > l \end{Bmatrix} \tag{16}$$

Let ξ_1 be the event that an alternate path has k or more occupied circuits and ξ_2 be the event that the alternate path has $k-1$ occupied circuits *and* more than $l-1$ busy channels on the less busy link. As ξ_1 and ξ_2 are mutually exclusive, $Z_{k,l} \equiv \text{Prob}\,[\xi_1 \text{ or } \xi_2]$ can be computed as

$$Z_{k,l} = \left\{1 - \text{Prob}\begin{bmatrix}\text{a link has less}\\ \text{than } k \text{ occupied}\\ \text{channels}\end{bmatrix}^2\right\} + \left\{\text{Prob}\begin{bmatrix}\text{the busier link has } k-1 \text{ occupied}\\ \text{channel and the other link has}\\ \text{more than } l-1 \text{ occupied channels}\end{bmatrix}\right\} =$$
$$\begin{Bmatrix} 1 & k = 0 \\ 1 - \left(\sum_{i=0}^{k-1} P_i\right)^2 & 1 \le k \le N \end{Bmatrix} + \begin{Bmatrix} 0 & k = l \\ 2\,P_{k-1}\left(\sum_{i=l}^{k-1} P_i\right) - P_{k-1}^2 & k > l \end{Bmatrix} \tag{17}$$

Moreover, given that alternate path ABC has k and l busy channels, let ξ_3 be the event that there are $\alpha - 1$ other alternate paths also having k and l busy channels and ξ_4 be the event that each of the remaining $m - \alpha$ alternate paths has either more than k occupied circuits *or* has k occupied circuits and more than l busy channels on the less busy link. Then, $f(\alpha \mid k, l) \equiv \text{Prob}\,[\xi_3 \text{ and } \xi_4]$ is given by

$$f(\alpha|k,l) = \binom{m-1}{\alpha-1} Y_{k,l}^{\alpha-1}\, Z_{k+1,l+1}^{m-\alpha} \tag{18}$$

$$A_i = 2m\sum_{j=0}^{N-1} P_j\, y(\max(i,j),\min(i,j)) = 2m\left[\sum_{j=0}^{i-1} P_j\, y(i,j) + \sum_{j=i}^{N-1} P_j\, y(j,i)\right]$$
$$= \begin{cases} \dfrac{DP_N}{P_i}\left[\sum_{j=0}^{i-1}\left(Z_{i+1,j}^m - Z_{i+1,j+1}^m\right) + 2\left(Z_{i+1,i}^m - Z_{i+1,i+1}^m\right) + \sum_{j=i+1}^{N-1}\left(Z_{j+1,i}^m - Z_{j+1,i+1}^m\right)\right] & 0 \le i \le N-2 \\ \dfrac{DP_N}{P_i}\left[\sum_{j=0}^{i-1}\left(Z_{i+1,j}^m - Z_{i+1,j+1}^m\right) + 2\left(Z_{i+1,i}^m - Z_{i+1,i+1}^m\right)\right] & i = N-1 \end{cases}$$
$$= \begin{cases} \dfrac{DP_N}{P_i}\left[Z_{i+1,0}^m + Z_{i+1,i}^m - 2\,Z_{i+1,i+1}^m + \sum_{j=i+1}^{N-1}\left(Z_{j+1,i}^m - Z_{j+1,i+1}^m\right)\right] & i = 0,1,\ldots,N-2 \\ \dfrac{DP_N}{P_i}\left[Z_{i+1,0}^m + Z_{i+1,i}^m - 2\,Z_{i+1,i+1}^m\right] & i = N-1 \end{cases} \tag{23}$$

Therefore, given k and l, the amount of traffic $y(k,l)$ that gets routed from AC to alternate path ABC is

$$y(k,l) = \sum_{\alpha=1}^{m} \frac{D\,P_N}{\alpha} f(\alpha|k,l) = D\,P_N \frac{\left(Y_{k,l} + Z_{k+1,l+1}\right)^m - Z_{k+1,l+1}^m}{m\,Y_{k,l}} \tag{19}$$

Therefore, given link AB has occupancy i, the overflowed traffic a_i from link AC to link AB is

$$a_i = \sum_{j=0}^{N-1} y\left(\max(i,j), \min(i,j)\right) P_j \quad i = 0,1,\ldots,N-1 \tag{20}$$

Since link AB carries the alternate traffic from $2m$ alternate paths, when it has i busy channels, the total alternate-route traffic A_i on it is

$$A_i = 2\,m a_i \tag{21}$$

As before, the call arrival and the call departure rates are

$$\begin{aligned} \lambda_i &= D + A_i & i &= 1,2,\ldots,N \\ \mu_i &= i & i &= 0,1,\ldots,N-1 \end{aligned} \tag{22}$$

To start the iterative solution of the state probabilities $\{P_i\}$, we observe that

Note that $Y_{i,j} + Z_{i+1,j+1} = Z_{i+1,j}$. As before, substituting (23) into the balance equation, $\{P_i\}$ can be solved recurrsively as in the last section.

3.2. *With trunk reservation*

With trunk reservation, the last r free channels on a link are always reserved for direct route traffic. Hence the call arrival and the call departure rates on a particular link become

$$\lambda_i = \begin{cases} D + A_i & 0 \le i \le N - r - 1 \\ D & N - r \le i \le N - 1 \end{cases} \quad (24a)$$

$$\mu_i = i \qquad i = 1, 2, \ldots, N \quad (24b)$$

where

$$A_i = 2\,m \sum_{j=0}^{N-r-1} P_j\, y(\max\,(i, j), \min\,(i, j)) \qquad i = 0, 1, \ldots, N - r - 1 \quad (25)$$

For $i > N - r$, we can solve the balance equation directly to obtain

$$P_i = \frac{(N - r)!\, D^{i-N+r}}{i!} P_{N-r} \qquad N - r + 1 \le i \le N \quad (26)$$

Substituting (26) into (25), A_i can be expressed in terms of $P_i, P_{i+1}, \ldots, P_{N-r}$. Further substituting into the balance equation, $\{P_i\}$ can again be solved recurrsively.

4. PERFORMANCE COMPARISONS

Fig. 1 shows the stationary state probability of M^2 routing with trunk reservation under different direct traffic loading. The truncated Gaussian form of the stationary state probability distribution is observed. As direct traffic increases, the dump bell curve shifts to the right, yielding a larger end-to-end blocking probability.

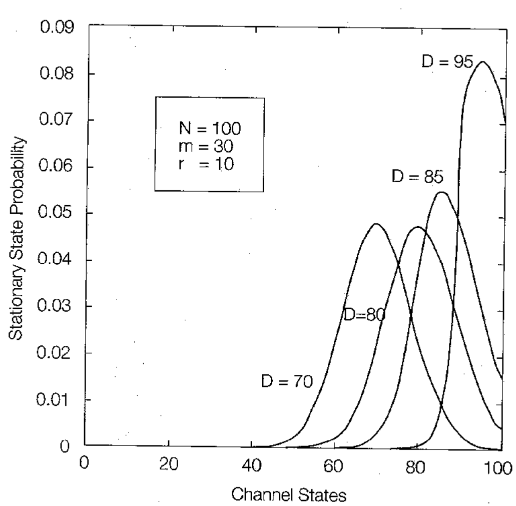

Fig. 1 - Stationary state probability of M^2 routing with TR.

Fig. 2 shows the alternate traffic rate of Random Alternate Routing (*RAR*) and M^2 routing with trunk reservation as a function of states for D equals 85, 90 and 95. We observe a sharp drop of A_i at a certain state and this drop becomes sharper as D increases. Comparing M^2 with *RAR*, we see that at moderate traffic load (say D = 85) M^2 routing has higher alternate traffic at lower states and smaller alternate traffic at higher states. This fact reflects the ability of M^2 to route alternate traffic to less congested alternate paths. We also observe that the percentage difference of alternate traffic rates between M^2 and *RAR* decreases with D. This shows that in heavy traffic conditions, the improvement on blocking of M^2 over that of *RAR* is not as significant as compared to that in moderate traffic conditions.

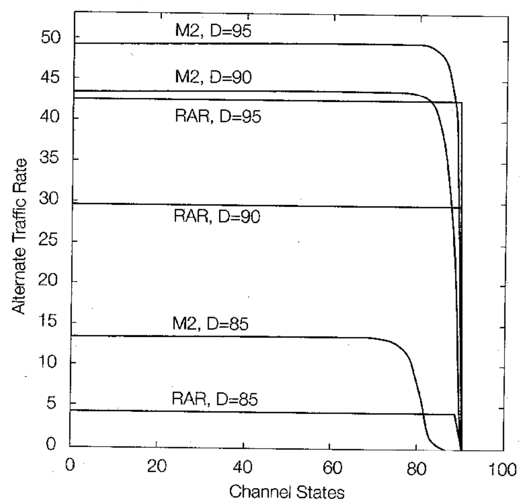

Fig. 2 - Alternate traffic rate vs states for M^2 routing and *RAR* with TR.

Fig. 3 shows the simulation results (those with markers) and the analytic results of the blocking probabilities of M^2 routing for various r values with N = 30 and D = 27. It is found that the analytic results match very well

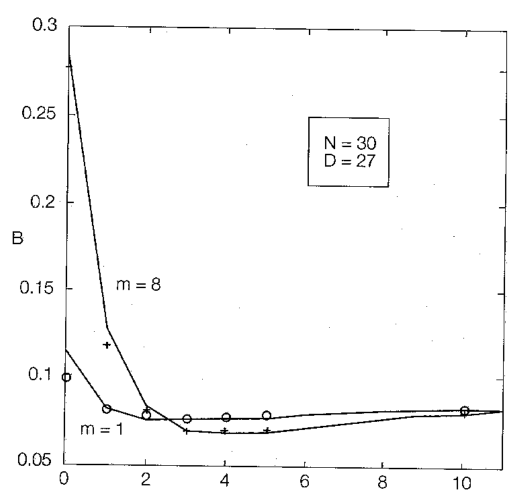

Fig. 3 - Blocking as a function of TR parameters, M^2 routing.

with the simulation results except for $r = 0$, where the results are a little bit off. Similar behavior was found for *RAR*, a plausible explanation was given in [14].

Fig. 4 a) shows the percentage improvement on the end-to-end blocking probability of M^2 routing over that of M routing as a function of D with $N = 10$ and $m = 6$. The M^2 routing has a property that its relative improvement over its counterpart depends on the direct traffic rate. A maximum of 30% and 16% relative improvements on the end-to-end blocking probability are observed for the case without and with trunk reservation. Fig. 4 b) shows the similar case for $N = 20$.

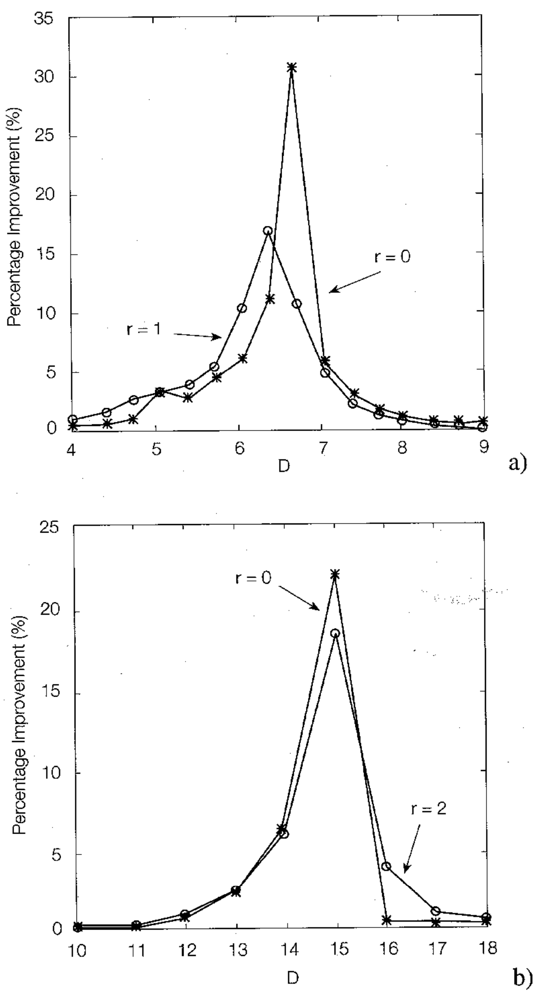

Fig. 4 a) Percentage improvement of M^2 over M. $N = 10$, $m = 6$: b) $N = 20$, $m = 10$, with and without trunk reservation.

Fig. 5 shows that for $D = 3$ and $N = 5$, the end-to-end blocking probability given by M^2 routing without trunk reservation is always smaller than that of M routing, independent of the network size. Similar behavior is found for other combination of D and N, and for the case with trunk reservation. It is also observed that without trunk reservation, the blocking increases with the number of alternate paths. This is also shown in Fig.

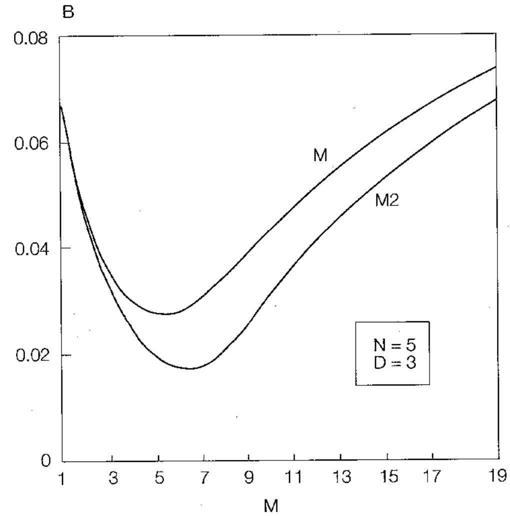

Fig. 5 - Blocking comparison of M^2 and M without *TR*.

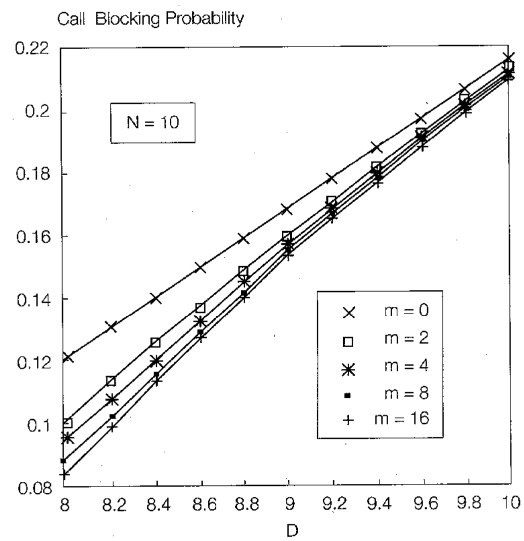

Fig. 6 - Blocking probability of M^2 with optimal r.

3 when $r = 0$, i.e., comparing B for $m = 1$ and $m = 8$.

Fig. 6 shows the end-to-end blocking probability of M^2 routing against direct traffic load for different number of alternate path using optimal trunk reservation parameters. Table 1 shows the optimal r values. It is seen that the optimal r increase with D and m. This figure shows that with the use of optimal r, the blocking probability decreases with increasing m. Hence, all available alternate paths in a network should be used provided that optimal r is also used.

Fig. 7 shows the percentage improvement of M^2 over M routing with trunk reservation for different values of r where $N = 10$, $D = 20/3$. We observed that the percentage improvement on blocking probability increases with m. This phenomenon is not found in the case without trunk reservation (Fig. 5).

Table 1 - *Optimal trunk reservation parameters*

	# of Alternate Paths			
D	$m = 2$	$m = 4$	$m = 8$	$m = 16$
8.0	1	2	2	2
8.2	1	2	2	2
8.4	2	2	2	2
8.6	2	2	2	2
8.8	2	2	2	2
9.0	2	2	2	3
9.2	2	2	2	3
9.4	2	2	3	3
9.6	2	3	3	3
9.8	2	3	3	3
10.0	3	3	3	3

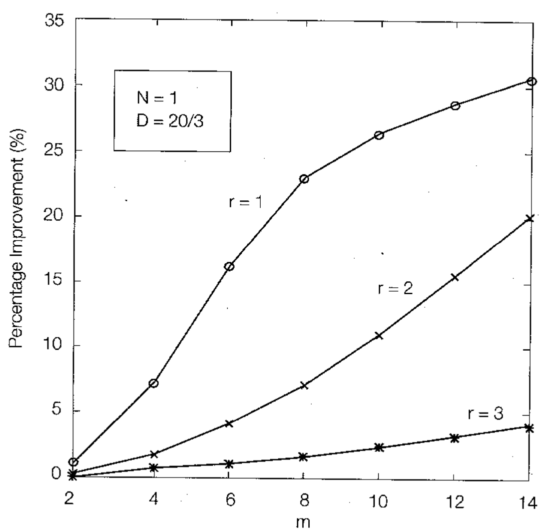

Fig. 7 - Percentage improvement in *B*, M^2 over *M* with TR.

5. CONCLUSIONS

We have analyzed the M and M^2 routings using a fixed point model where the rate of the alternate traffic offered to a link depends on the state of the link. The M^2 routing is found to provide a small but significant improvement over M routing when the number of alternate paths is large and/or the trunk group size is small. As the implementation of M^2 routing is no more complicated than M routing (both requiring the same channel occupancy information) and its performance is always better than M routing, M^2 routing is deemed a better rule to use.

We have also studied the performance of the reversed M^2 routing, i.e., the rule that chooses an alternate path with minimum occupancy first, and if there is a tie, choose one with the maximum number of free circuits. Extensive simulation on a 9-node fully connected symmetric network shows that the end-to-end blocking probability is virtually the same as that for M^2 routing under moderate to heavy traffic conditions. More study is needed to explain why this is so. Other state-dependent rules can be formulated with different uses of the channel occupancy information and more elaborate routing rules should also take the traffic rates into consideration.

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