LCR: Local Collaborative Ranking

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Matrix Completion Problem

- Problem: given a partially-observed noisy matrix M, we would like to approximately complete it.

- Application: recommendation systems
  - $M_{u,i}$ is rating of item i by user u.
  - Naturally sparse: most are unknown.
  - We want to estimate unrated items.

Items

Users					
			3		
	3				5
		5		4	
			2	5	
	1			2	

Low-rank Assumption

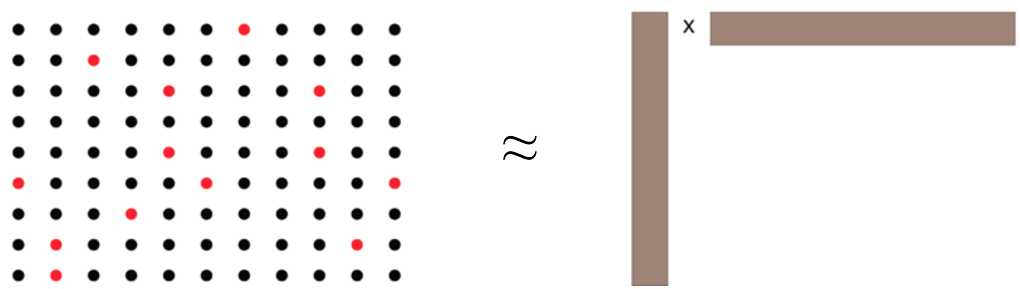
- Common practice: low-rank assumption.

$$M \approx UV^T \in \mathbb{R}^{n_1 \times n_2}, \qquad U \in \mathbb{R}^{n_1 \times r}$$

$$V \in \mathbb{R}^{n_2 \times r}$$

$$r \ll \min(n_1, n_2)$$

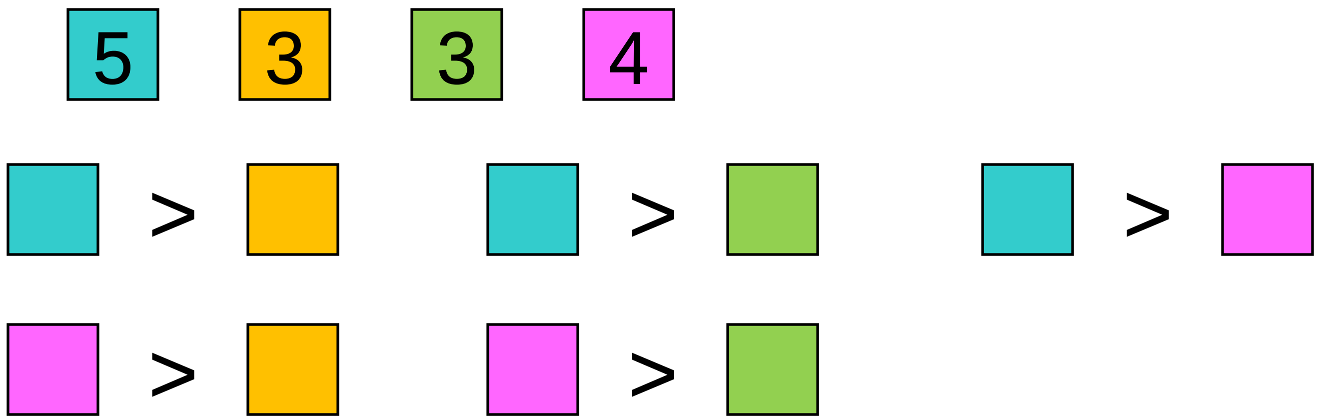
- Incomplete SVD: $\min_{U,V} \sum_{(u,i) \in A} \left([UV^T]_{u,i} - M_{u,i} \right)^2$



● Observed ● Unobserved

Ordering Problem

- Motivation: we usually care about relative order of preference, not exact score.
- Order items according to the (partial) preferences of a given user.
- Example: for the following user who rated 4 ratings,



Talk Agenda & Contribution

- Paired loss functions
  - How to solve ordering problem?
- Local Low-Rank Assumption
  - Why and how to tackle diminishing returns?
- Algorithm
  - Should be scalable for big data.
- Experimental analysis
  - Two frameworks.

Ordering Function

- Learn an ordering function f , such that $f(u, i) > f(u, j)$ if $M_{u,i} > M_{u,j}$.
  - Not necessarily $f(u, i) \approx M_{u,i}$.

- Pair-wise Loss function $L(\Delta M, \Delta f)$

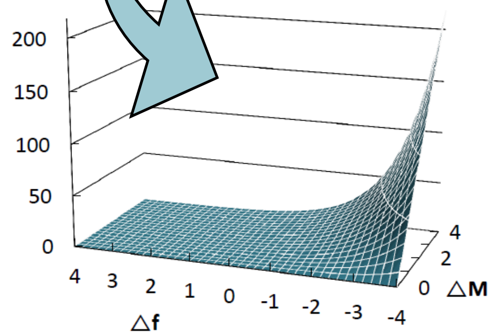
$$E(f) = \sum_{u} \sum_{(i,j) \in M_u} L(M_{u,i} - M_{u,j}, f(u,i) - f(u,j))$$

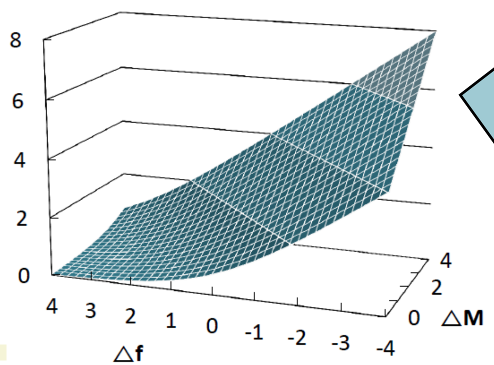
  - $\Delta M = M_{u,i} - M_{u,j}$: difference of observed ratings.
  - $\Delta f = f_{u,i} - f_{u,j}$: difference of estimated ratings.

Pair-wise Loss $L(\Delta M, \Delta f)$

- Zero-one loss
  - Assigns (same) positive loss when $\Delta M \, \Delta f < 0$.
  - Not differentiable.
-

	Multiplicative	Additive
Log-loss	$\Delta M \log(1 + e^{-\Delta f})$	$\log(1 + e^{\Delta M - \Delta f})$
Exp-loss	$\Delta M \exp\{-\Delta f\}$	$\exp\{\Delta M - \Delta f\}$
Hinge-loss	$\Delta M \, [-\Delta f]_+$	$[\Delta M - \Delta f]_+$

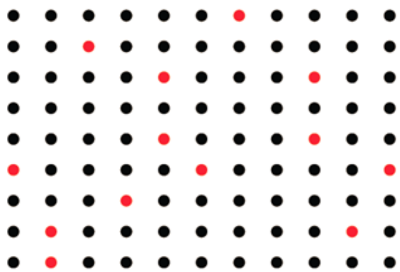




Global Approximation

- With $f(u, i) = [UV^T]_{u,i}$, solve matrix factorization problem with respect to a paired loss L.

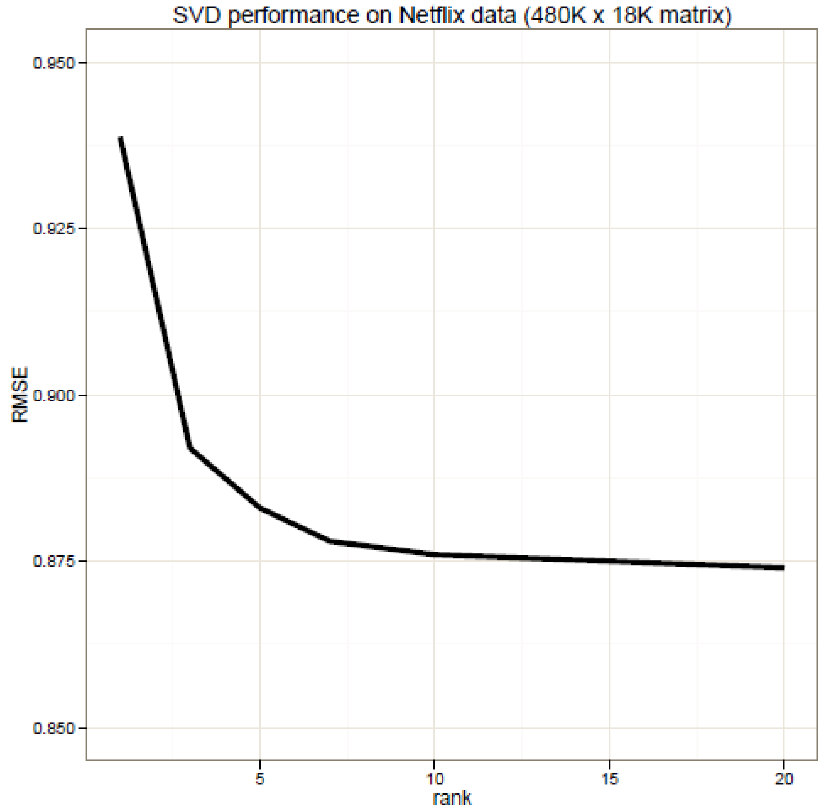
$$\min_{U,V} \sum_u \sum_{(i,j)\in M_u} L(M_{u,i} - M_{u,j}, [UV^T]_{u,i} - [UV^T]_{u,j})$$

- We model  using  form,

  so as to minimize a pair-wise loss.

Diminishing Returns

- Small improvement as capacity increases.



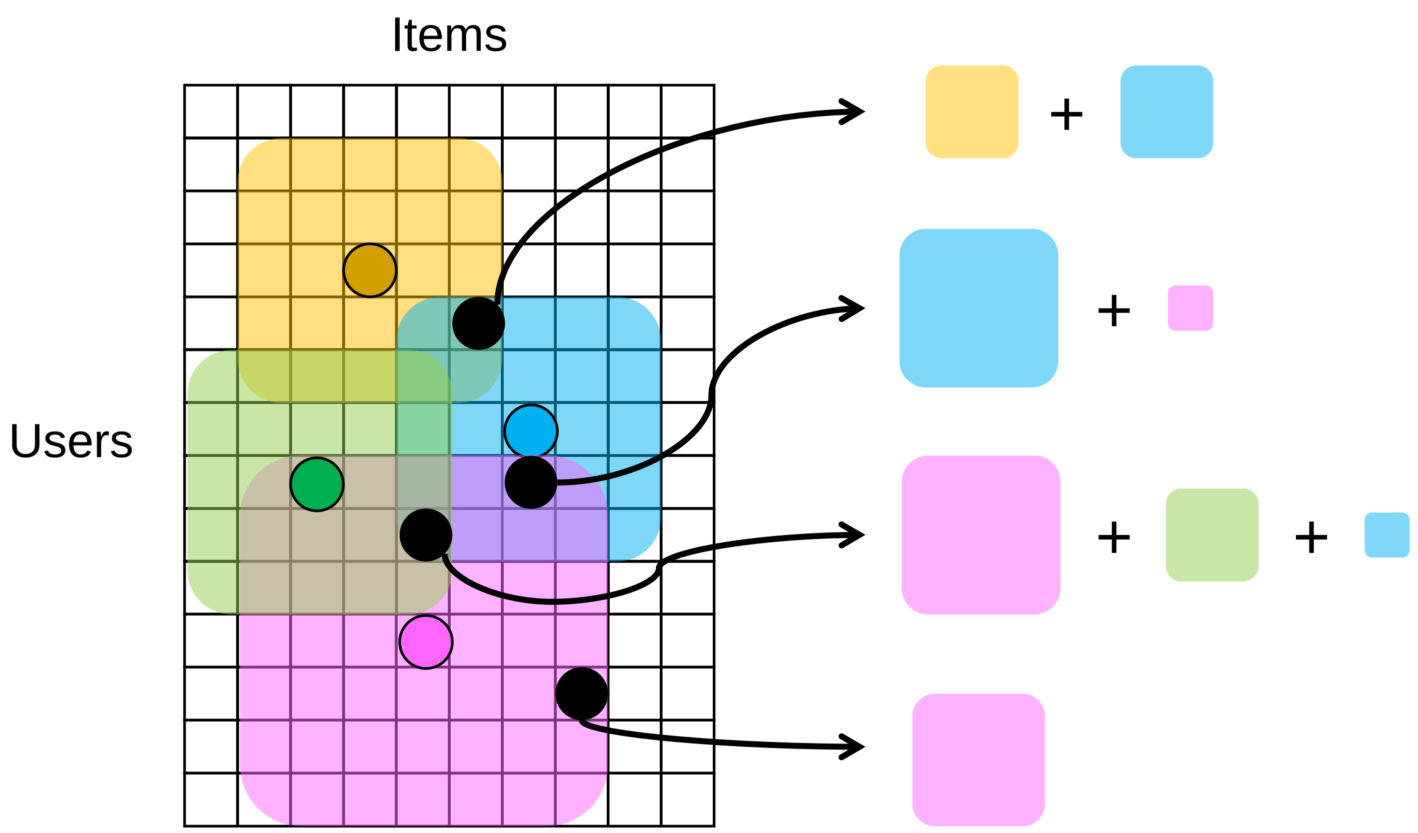
Why diminishing returns?

- Hypotheses
  - **H1**: M has low rank; it reflects best possible prediction.
  - **H2**: M has high rank; diminishing returns due to over-fitting, or convergence to a poor local optimum.

- In recommendation systems,
  - **H2** is a realistic assumption.
  - **H1** is unrealistic globally, but it's realistic locally.

- The rating matrix is only locally low-rank.
  - Low-rank only with subset of similar users and items.

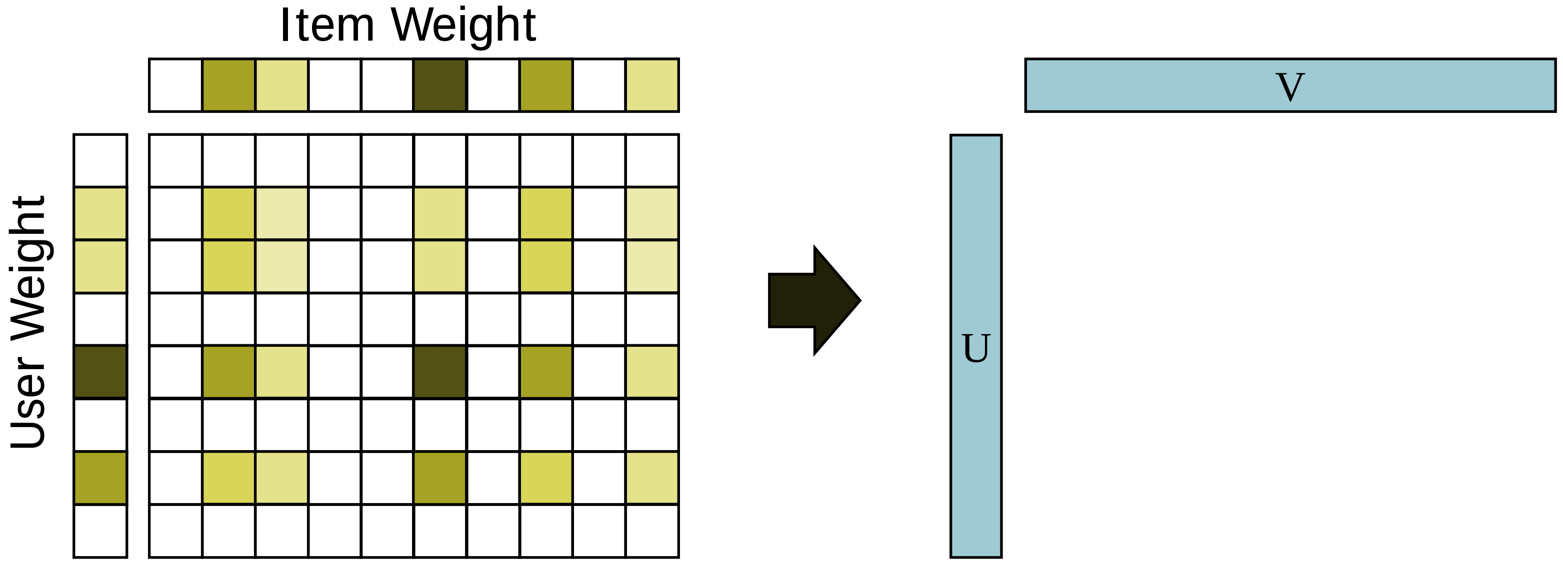
Local Low-rank Matrix Approx.



[Lee et al, 2013 ICML]

Learning Algorithm

- Run in Parallel:
  - Step 1: Select an anchor point.
  - Step 2: Calculate user/item weight using kernel smoothing.
  - Step 3: Solve a weighted matrix factorization problem.



Evaluation

- Goal: Recommend most preferable items based on precise estimation of order of preference.

- Criteria
  - Zero-One Error: the ratio of correctly ordered test pairs.
  - Average Precision: the ratio of preferred items in the list.
  - NDCG@k: optimality of the order of recommendation list.

- Dataset: MovieLens, EachMovie, Yelp

Data Split

- Fixed ratio
  - For each user, 50% of ratings are used for training, rest of them are for testing.
  - More realistic: take cold/cool-start users into account.

→ Used <u>to see effects of parameters</u>.

- Fixed number
  - Users with more than 20 ratings are considered. 10 ratings are used for training, and rest of them are for testing.
  - More stable: consider users with sufficient ratings only.
  - Widely used in literature with N=10.

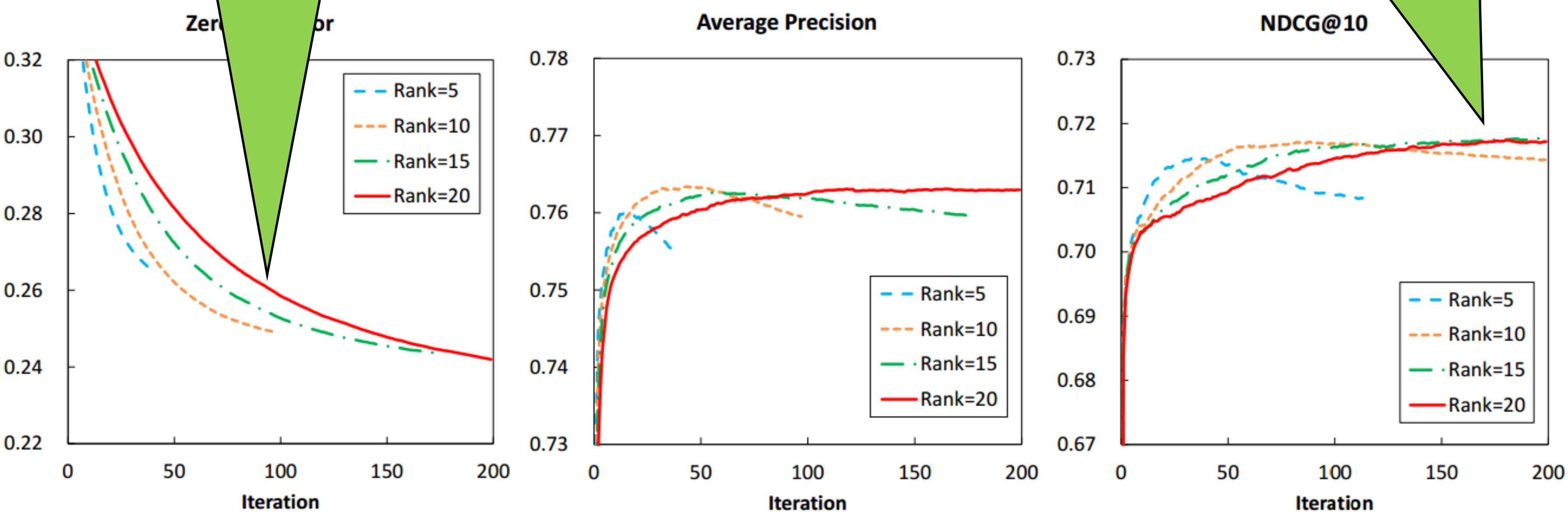
→ Used <u>to compare with existing methods</u>.

Effect of Capacity

<u>Zero-one</u>: With higher dimension, converges slowly.

<u>All</u>: With higher dimension, ultimate performance is better.

<u>AvgP, NDCG</u>: With higher dimension, it less overfits.

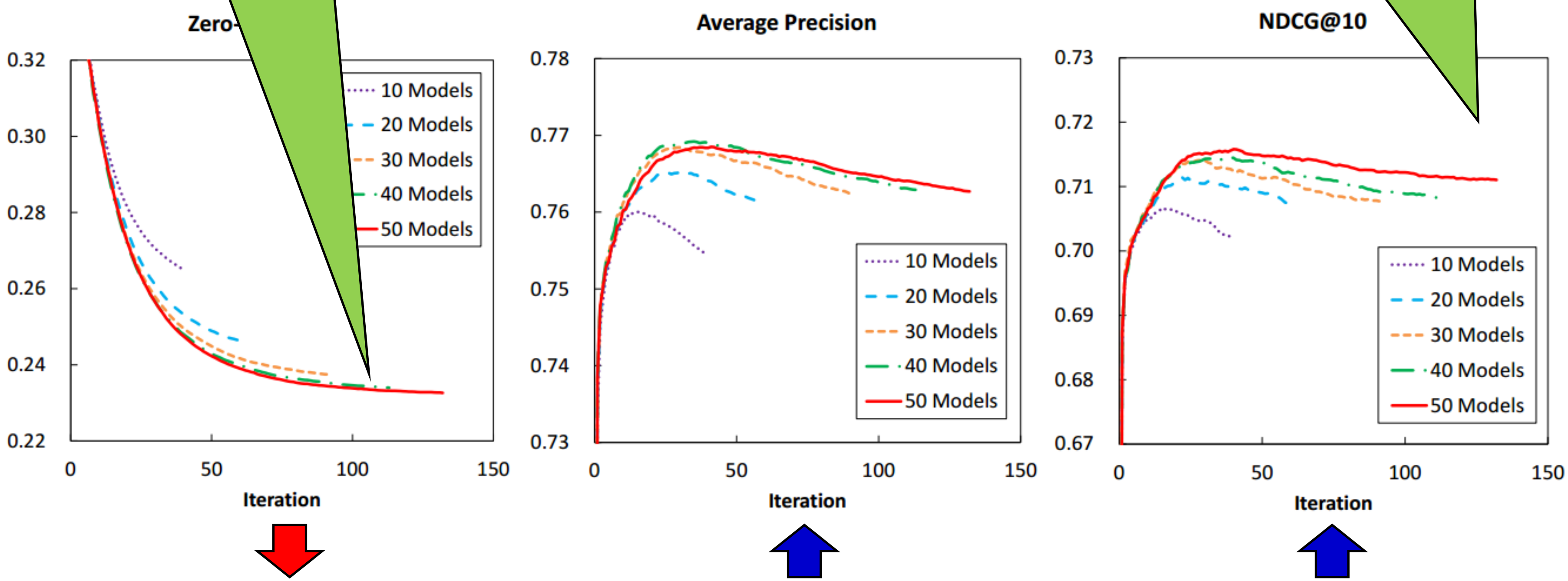


Effect of Number of Local Models

Zero-one: With more local models, converges slowly.

All: With more local models, ultimate performance is better.

AvgP, NDCG: With more local models, it less overfits.



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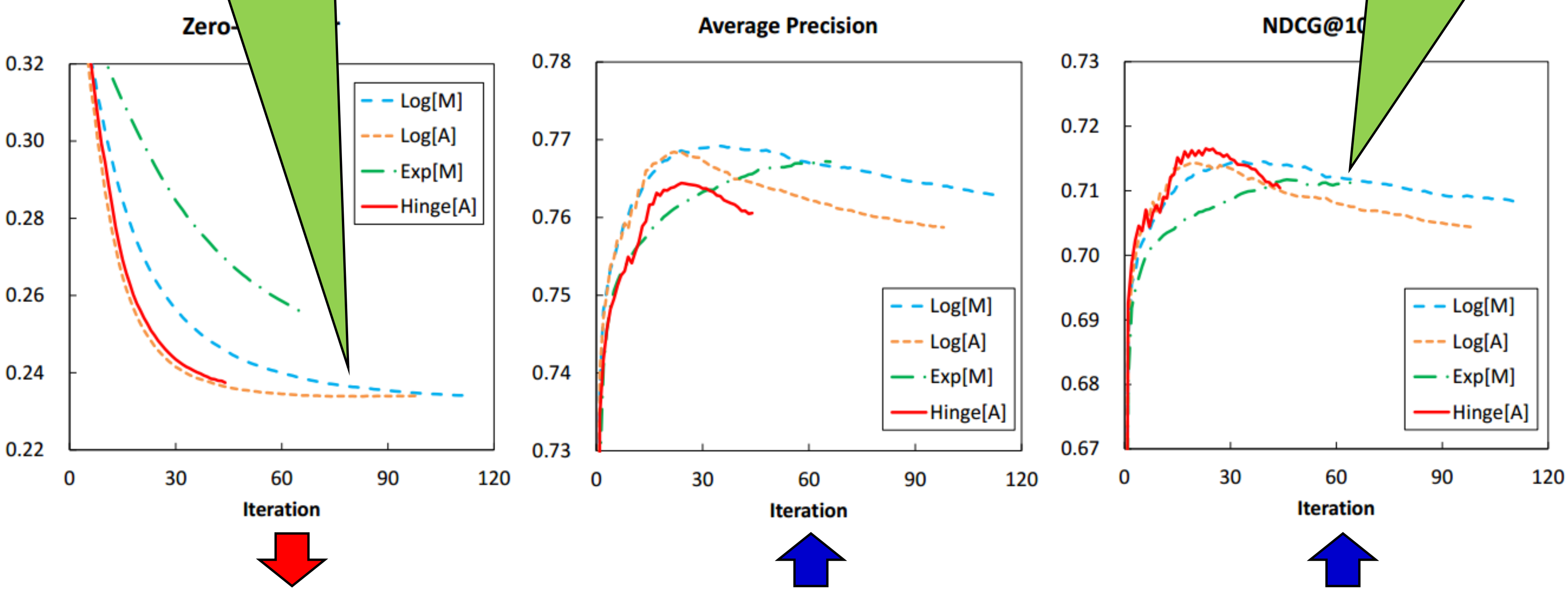
Effect of Loss Functions

Zero-one: Hinge[A], Log[A] performs best.

All: Convergence and overfitting depends on loss function.

AvgP, NDCG: Log[M], Exp[M] performs best.



Comparison with other methods

	Method	Average Precision	NDCG@10
MovieLens	CofiRank	0.6632	0.6502
	GCR (SVD with ranked loss)	0.7209	0.6990
	LCR	**0.7406**	**0.7152**
EachMovie	CofiRank	**0.7491**	0.6635
	GCR (SVD with ranked loss)	0.7088	0.6998
	LCR	0.7307	**0.7166**
Yelp	CofiRank	0.7246	0.6997
	GCR (SVD with ranked loss)	0.7754	0.7465
	LCR	**0.7903**	**0.7575**

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Take-home Messages

- In recommendation systems, the rating matrix is low-rank only locally.
- Local low-rank assumption is realistic for ordering problem as well as rating prediction.
- LCR (Local Collaborative Ranking) algorithm is highly parallelizable and scalable.

Source code available soon!

- PREA toolkit: **http://prea.gatech.edu**