The 11th Joint International Computer Conference (JICC2005)

COMPARATIVE STUDIES ON THE CT IMAGE RECONSTRUCTION BASED ON THE RBF NEURAL NETWORK*

HU Ming[1], GUO Ping[1,2], LYU Michael R.[2]

[1]*Department of Computer Science, Beijing Normal University, Beijing, 100875, China*
E-mail: pguo@bnu.edu.cn

[2]*Department of Computer Science & Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong, China*
E-mail: lyu@cse.cuhk.edu.hk

Abstract: To reconstruct two-dimensional computerized tomography (CT) images from a small amount of projection data is a very difficult task. In this paper, two methods based on radial basis function (RBF) neural network are investigated to perform such a work. In the first method, we take projection data as the input and original image as the output of the network, after trained with some samples, the network can be applied to reconstruct CT image in the same class. In the second method, we adopt coordinate and a cross-section image as the input and the output respectively. For converting the image to its projections, an additional integral module is cascaded with the network. To evaluate these two methods, a comparative study is presented. A pixel-wise error estimator is adopted to calculate the overall error of the reconstructed images. Experiments show that the second method is the best for moderate projection data in practice.

Keyword: CT image reconstruction; RBF neural network; projection data.

1. Introduction

The problem of computerized tomography (CT) image reconstruction has become important during the last decades. Though in some medical fields the techniques attain a sophisticated level of completion, there are still many issues to be solved or improved. One of them is to develop a method of an image reconstruction from data of a small amount of the projection paths [1][2].

In practice, there often occur the problems with insufficient sets of projection data. Traditionally, Algebraic Reconstruction Technique (ART) can be adopted need to solve these problems [5]. But its applied domain is limited because of some deficiencies, such as a relatively high time complexity, the demand for distribution of the projection paths, etc. Therefore it is worthy for us to investigate more effective new methods.

The RBF neural network can be considered as a nonlinear function, which maps a point in a R^n input data space into a point in a R^m output data space. The procedure to determine the parameters of the network from given data sets of the input-output relations is called learning [3][4]. As interpolation and smoothing processes are inherently included in the learning process, the above problem of the CT image reconstruction for a small amount of the projection data are relaxed considerably. In this work, we investigate two methods based on RBF neural network to practical problems. To evaluate these two methods, we also compare them with traditional ART method under a same criteria.

2. CT Image Reconstruction Algorithm

2.1. *Method One*

We propose this new method to reconstruct medical CT medical image. In this method, the input data sets are composed with $K \times L$ dimension vectors, which are transformed from projection data of samples, and the output data sets are with $M \times N$ dimension vectors of the image samples. So the network output stands for a sample image, and the corresponding input data expresses projection of the sample image. The network is trained with some samples, which come from a same class of CT image. The parameters of the network can be determined during a network learning process [1]. The trained network can be applied to reconstruct CT image.

The structure of the RBF neural network is shown in Fig. (1). The Gaussian function is selected as basis function for RBF networks, which is given by

$$G_i(r_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp[-\frac{r_i^2}{2\sigma_i^2}] \ . \tag{1}$$

* This work is supported by a grant from the NSFC (60275002) and a grant from the HK RGC (CUHK4182/03E).

Where r_i is the Euclidian distance between the image vectors and the hidden neural vectors. It is defined as

$$r_i = ||X - C_i||. \qquad (2)$$

And σ_i and C_i are a standard deviation and the center location of the i-th function respectively [2][3]. During the learning process, the pseudo-inverse algorithm is adopted to estimate the parameters of the network [1][4].

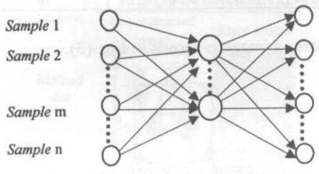


Figure 1. RBF network structure

2.2. *Method Two*

In this method, the cross-sectional image is represented by a RBF network, the image reconstruction is achieved from the learning process of the network. The learning process is conducted based on the projections of the image instead of the image itself.

The structure of the RBF network with integral module is shown in Fig. (2). For this network, we adopt coordinate as the input and a cross-section image as the output, respectively. An additional integral module is added end on the RBF network output for converting the image to its projections [3]. Total error is given in Eq. (3). Applying a gradient descent rule to the defined error, the network gets close to a local minimum error, which means that the images are adequately reconstructed.

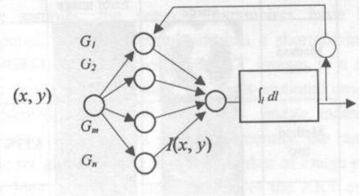


Figure 2. RBF network structure

Here, the network plays a role of functional mapping an image coordinate (x, y) to its intensity $I(x, y)$ as an output. r_i is the Euclidian distance between the image coordinate $X(x, y)$ and the i-th center position C_i of basis function. In the network, Gaussian functions-$G_i(r_i)$ given in equation (1) are used as basis functions,.

Total error is defined as:

$$E = \frac{1}{2}\sum_{m=1}^{M}(r_m - g_m)^2 \qquad (3)$$

The output layer implements a weighted sum of the hidden-unit outputs, which can be computed as

$$I(x,y) = \sum_{i=1}^{N} w_i G(x,y) \qquad (4)$$

$$= \sum_{i=1}^{N} w_i \frac{1}{\sqrt{2\pi}\sigma_i}\exp\left[-\frac{\|X - C_i\|}{2\sigma_i^2}\right]$$

So, the projection of the m-th output is a line integral along the m-th path obtained numerically by using the output of the neural network as Eq. (5).

$$g_m = \int_{L_m} I(x,y)dl = \sum_{i=1}^{N} w_i H_i^m(x,y) \qquad (5)$$

$$H_i^m(x,y) =$$

$$\int_{L_m} \frac{1}{\sqrt{2\pi}\sigma_i}\exp[-\{(x-c_x^i)^2 + (y-c_y^i)^2\}/2\sigma_i^2]dl_m$$

In the following we will introduce the rules of updating the parameters of the network.

Weight updating process: the current weights $w_i(t)$ of the network are update to be $w_i(t+1)$ in the next step using the gradient descent rules, as shown in Eq. (6) .

$$w_i(t+1) = w_i(t) - \eta\frac{\partial E_{total}}{\partial w_i}$$

$$i = 1, 2, \ldots, N \qquad (6)$$

$$\frac{\partial E_{total}}{\partial w_i} = -\sum_{m=1}^{M}(r_m - g_m)H_i^m(x,y)$$

Standard deviation of the radial basis function updating process is show in the following equation.

$$\sigma_i(t+1) = \sigma_i(t) - \varsigma\frac{\partial E_{total}}{\partial \sigma_i}$$

$$i = 1, 2, \ldots, N \qquad (7)$$

$$\frac{\partial E_{total}}{\partial \sigma_i} = -\sum_{m=1}^{M}(r_m - g_m)\frac{\partial g_m}{\partial \sigma_i} =$$

$$-\sum_{m=1}^{M}(r_m - g_m)w_i\frac{\partial}{\partial w_i}H_i^m(x,y)$$

In Eq. (6) and Eq. (7), the $H_i^m(x,y)$ is unknown, but it can be approximated according to Eq. (5). We compute it as follow.

The projection ray can be expressed by Eq. (8).

$$L_m : y = k_m x + b_m . \tag{8}$$

We assume that the object is located only within a reconstruction area, and thus density value outside the area is totally zero, the integral of infinite range should be almost the same as that of reconstruction area [6]. Thus, the matrix H can be obtained from

$$\begin{aligned} H_i^m(x,y) &= \\ &\sqrt{1^2+k_m^2}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_i}\exp[-\{(x-c_x')^2 \\ &+(k_m x+b_m-c_y')^2\}/2\sigma_i^2]dx \\ &=\exp\left[-\{(k_m c_x'+b_m-c_y)/2\sigma_i^2/(1+k_m^2)\}^2\right] \\ &\times\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_i}\exp\left[-x'^2/2\sigma_i^2\right]dx' \\ &=\exp\left[-\{(k_m c_x'+b_m-c_y)/2\sigma_i^2/(1+k_m^2)\}^2\right] \end{aligned} \tag{9}$$

Where

$$\begin{aligned} x' &= \sqrt{1^2+k_m^2}\,x \\ &-(k_m b_m - k_m c_y' - c_x')/\sqrt{1^2+k_m^2} \end{aligned} \tag{10}$$

3. Results Analysis

Two new methods based on the RBF network are employed to reconstruct CT image from a small amount projection data. In the experiments, the two methods are investigated from different points of view.

To compare the two methods with ART algorithm, we select a pixel-wise error estimator as the criteria, which is written in Eq. (11). The estimator can be used to evaluate the quality of the reconstructed image.

$$\delta_e = \sqrt{\frac{\sum_{x=1}^{width}\sum_{y=1}^{width}(f_{org}(x,y)-f_{rec}(x,y))^2}{\sum_{x=1}^{width} f_{org}(x,y)^2}} \tag{11}$$

where

$f_{org}(x,y)$: the pixel of the original image.

$f_{rec}(x,y)$: the pixel of the reconstructed image.

It is obvious the δ_e can evaluate the difference between the reconstructed image and the original image. δ_e increases if the difference between $f_{org}(x,y)$ and $f_{rec}(x,y)$ increases.

In the experiments, the original image is obtained from CT image database on the internet, and the projection data are obtained from a fan beam CT. The original image and projection description are shown in Table 1.

Table 1. Image and projection description

Image size	angel number of projections(a)	angel number of projections(b)
32×32	8	16

The origin image is shown in Fig. (3).

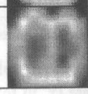
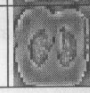
Figure 3 Original image

To compare the reconstructed image with the original image, we transform the difference between them into an error image, which is shown on the right of reconstructed images in the Table 2 and Table 3. The corresponding reconstructed image is on the left.

In our experiments, we investigated three methods including the two RBF network methods (Method one and Method two) and the ART (Method three). To compare the reconstructed image on different conditions. We implement the experiments under two kinds of conditions, as shown in Table 1.

In the first experiment, The projections data are achieved through eight projection paths with equal space between 0 and 180. The experiment result is shown in Table 2:

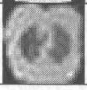
Table 2 The difference, original image and δ_e

Method	8 projection directions(a)		δ_e
	Reconstructed image	Error image	
Method one			12.87%
Method two			8.13%
Method three			23.84%

From this experiment, δ_e shows that the better reconstructed result are those two methods based on RBF network. By examining on the reconstructed image and error image, we can get the same conclusion. In three methods, the second method is best on the reconstructed precision.

In the second experiment, there are sixteen projection paths, results are shown in Table 3.

Table 3 The difference, original image and δ_e

Method	16 projection directions(a)		δ_e
	Reconstructed image	Error image	
Method one			11.52%
Method two			3.83%
Method three			15.64%

When analyzing Table 3 only, we can find the regularity is the same as the first experiment. But when we study the results and compare the second experiment to the first one, we can obtain more information. It is obvious that the δ_e in the first experiment is small than that in the second. We can know that the more projection data there are, the more precise the reconstructed image will be. The second method and ART is notable influenced by the number of projection data, but the Method one is not.

By analyzing the time complexity of the three methods, we find that Method one can reconstruct CT image very quickly. The reason is that after trained by some samples, the network parameters have been computed. But the Method one has a shortcoming: it only can be used to reconstruct CT images in a same class. In the second method, the cross-sectional image is represented by a RBF network, image matrix is converted to the weight vector. Generally, the number of the weight is smaller than the number of image pixel. Thus Method two has advantage over the ART on the memories and the time complexity. To sum up, the second method is feasible for CT image reconstruction.

4. Conclusions

In this paper, we comparatively studied two efficient image reconstruction method based on the RBF network in a limited view tomography. These two methods have several advantages over the conventional ART method. First, the cross-sectional image can be expressed with less number of elements compared to that of pixels in the method two, and in the method one, we can reconstruct the image by the trained network. These make reduction of the time in computation and the memories. Second, these two RBF network methods relax the demand for distribution of the projection paths; we can reconstruct the CT image from insufficient sets of projection data.

But there are some shortages in these two methods. If these two methods are employed to reconstruct the simple and relatively small images, they show advantage over the traditional ART. But if the object images are too complexity, the superiority is not obvious. Furthermore, the first method only can be used to reconstruct CT images in a same class. We will engage to develop more general techniques in the future research work.

References

[1] Fath El Alem, F. Ali, Zensho NaKao, and Yen-Wei Chen, "Two New Neural Network Approaches to Two-dimensional CT Image Reconstruction," Fuzzy Sets and Systems, vol. 103, pp. 295–302, 1999.

[2] Xiao Feng Ma, Makoto Fukuhara, and Tatsauoki Takeda, "Neural network CT image reconstruction method for small amount of projection data," Nuclear Instruments and Methods in Physics Research, vol. A499, pp. 366–377, 2000.

[3] Y. J. Roh and H. S. Cho, "Image Reconstruction in X-ray Tomography Using a Radial Basis Function (RBF) Neural Network," Optomechatronic System II, Proceeding of SPIE, vol. 4564, pp.35-46, 2001.

[4] P. Guo and Michael R. Lyu, "A Pseudoinverse Learning Algorithm for Feedforward Neural Networks with Stacked Generalization Application to Software Reliability Growth Data", Neurocomputing, vol.56, pp.101-121, 2004.

[5] G. T. Herman, Image Reconstruction from Projections. San Francisco: Academic Press, 1980.

[6] Z. Nakao, Y. Kina, and Y.W. Chen, "CT image reconstruction by simulated annealing from a small number of projections," in the 15th LASTED Int. Conf. on Applied Informatics, 1997, pp. 342–345.