ENGG2430: Homework # 1

Due: Jan 20, 2014, 5:00 PM

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1. We have three coins: two nickels (5 cents coin) and one dime (10 cents coin). Each coin has two sides: one is head and the other side shows the worth of the coin. You pay \$0.1 to enter the game to toss these three coins. You will receive those coins which fall heads up.
   (a) What is the sample space $\mathcal{S}$ of this problem?
   (b) What is the probability of each sample point in $\mathcal{S}$?
   (c) What is the probability of winning some money?
   (d) What is the probability of losing some money?
   (e) What is the probability of breaking even?
2. We keep tossing a coin until it lands head up.
   (a) What is the sample space $\mathcal{S}$?
   (b) What is the probability of only tossing the coin once?
   (c) What is the probability of only tossing the coin twice?
3. We want to count the number of car accidents in Hong Kong in a year. What is the sample space $\mathcal{S}$?
4. I see an opened can of coca-cola, and want to find out the probability that it is less than $1/3$ filled. What is the sample space $\mathcal{S}$ of this problem?
5. Let $A = \{1, 2, 3, 4, 5\}$. Consider A_3 is a 3-element subset of A.
   (a) What are all possible elements of A_3?
   (b) If A has n distinct elements, what is the size of the subset A_k, where $k \leq n$?
6. Four people are randomly seated in a row, we want to find the probability that you are sitting beside your girlfriend (or boyfriend).
   (a) What is the sample space $\mathcal{S}$?
   (b) What is the probability you are sitting next to your love one?
   (c) Instead of four people, we have ten people sitting in a row. What is the probability you are sitting next to your love one?

(d) Instead of ten people sitting in a row, we have ten people sitting in a *circle*.

- What is the state space $\mathcal{S}$ of this problem?
- What is the probability you are sitting beside your love one?

7. Assume in the CSE Department, the probability that a computer is of Window OS or Mac OS is equally likely. You have three computers in front of you.

   - What is the state space $\mathcal{S}$ of this problem?
   - What is the probability you have exact one computer with Mac OS?
   - Assume that the probability that a computer is of Window OS is 0.9 and Mac OS is 0.1. You have three computers in front of you, what is the probability of having exactly two computers with Mac OS?

Challenging but bonus problems

1. Bob needs to reach his sweetheart Alice. As illustrated in the figure, both of them are located in an m by n city grid. Bob is located in the lower left hand corner while Alice is located in the upper right hand corner. Bob is smart and he only wants to travel through the *shortest path*.

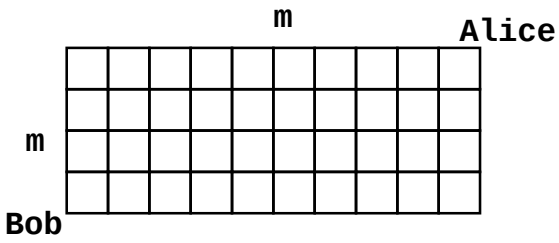

Figure 1: Bob needs to reach Alice

   - If $m = 2; n = 3$. How many shortest paths are there for Bob to choose from?
   - What is the characteristics of these shortest paths?
   - If $m \geq 1$ and $n \geq 1$, how many shortest paths are there in an m by n grid for Bob to choose from?

2. There are k black balls and a single white ball in an urn. Bob and Alice draw balls from the urn without replacement. The game is won by the person who happens to draw the single red ball. Bob is a gentleman and he offers Alice the *choice* to choose to start first or not. Alice has a hunch that she might be better off if she starts because she might succeed in the first draw. However, if her first draw results in a black ball, then Bob's chance to draw the red ball in his first draw are increased because there is one less black ball in the urn. Show Alice choose to start first or not so as to maximize her chance of winning?

- There are two possibilities: k is even or k is odd. Assume k is odd first.
  - What is the sample space S?
  - What is the probability that Alice will win in her first draw?
  - What is the probability that Alice will win in her second draw?
  - What is the probability that Alice will win? What is the probability that Bob will win?
- Repeat the same analysis when k is even.
- If you are Alice, what is your *correct* strategy in choosing go or no go?