Stochastic Analysis and File Availability Enhancement for BT-like File Sharing Systems

John.C.S.Lui

The Chinese University of Hong Kong

Outline

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- **Introduction**

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- Introduction
- Modeling the Performance

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- Modeling the Performance
- Extension on heterogeneous Network

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- Conclusion

The Problem with C/S Publishing

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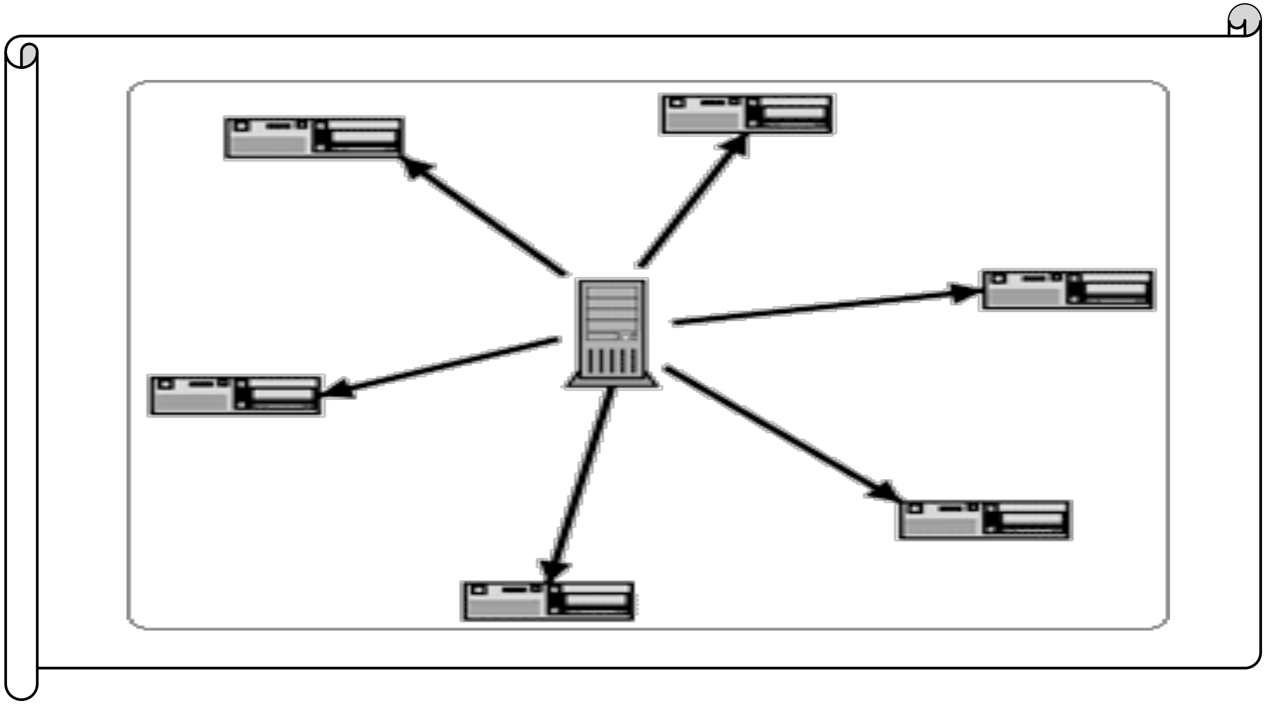
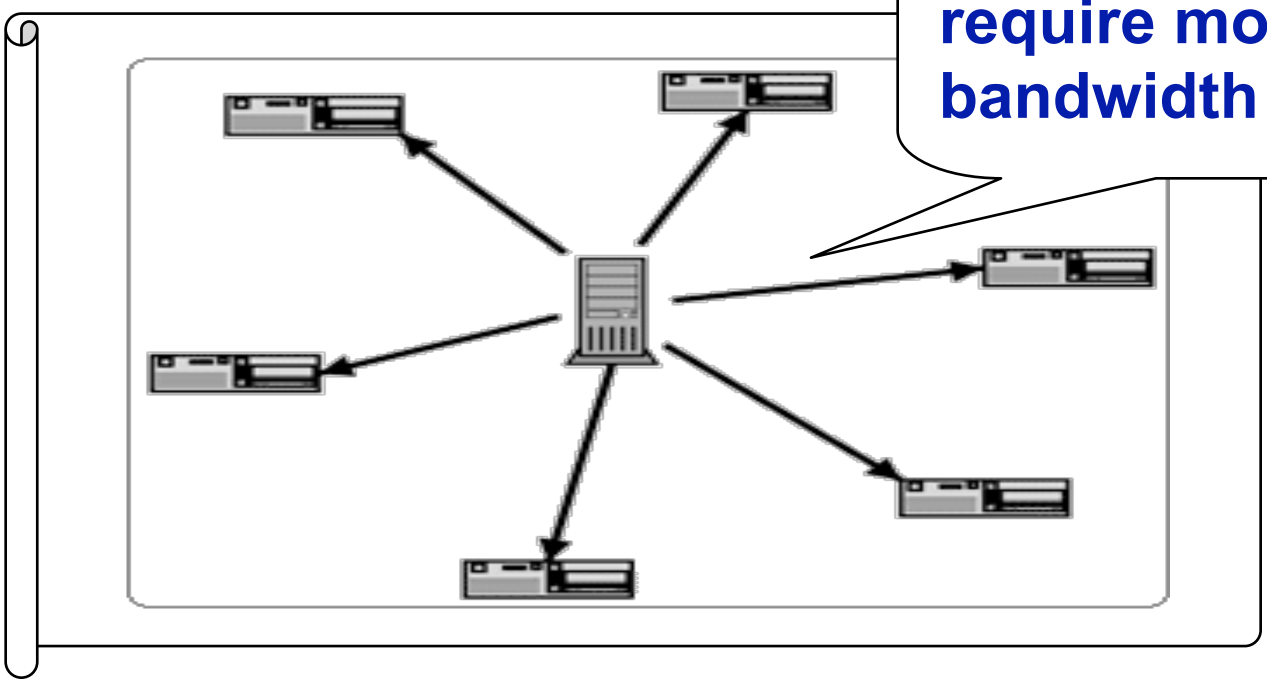


The Problem with C/S Publishing

More customers
require more
bandwidth



BitTorrent Solution

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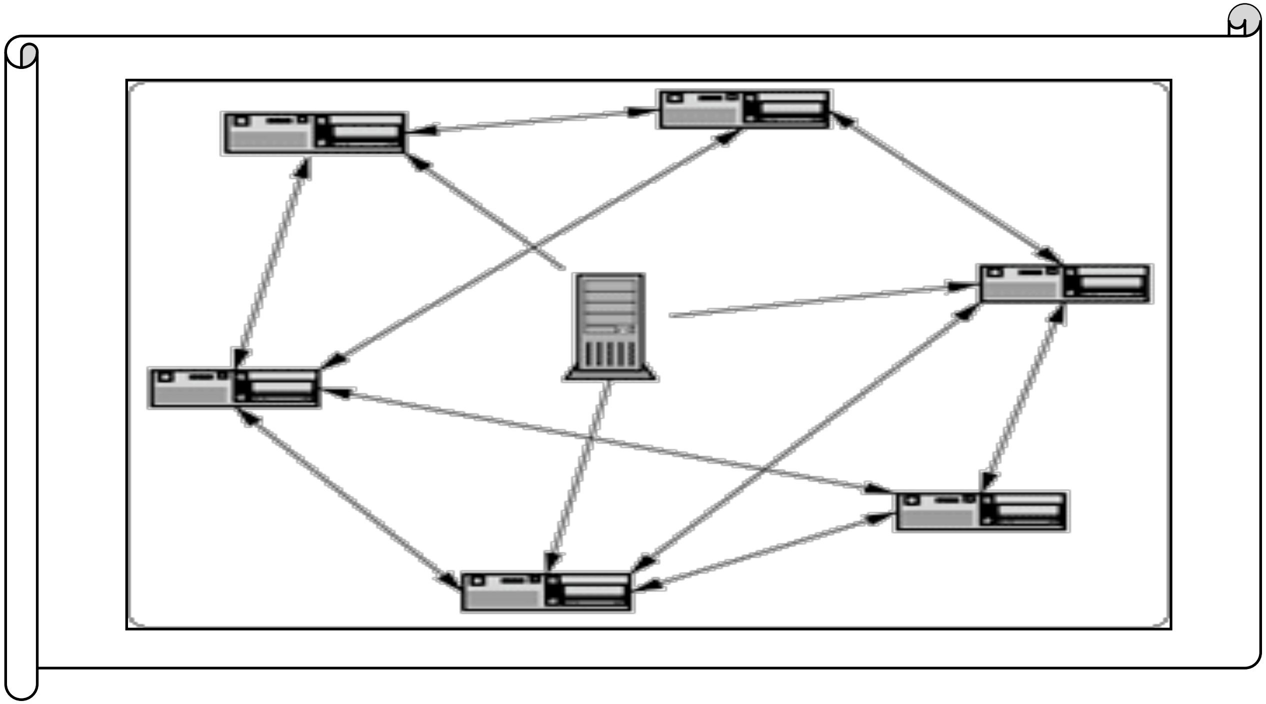
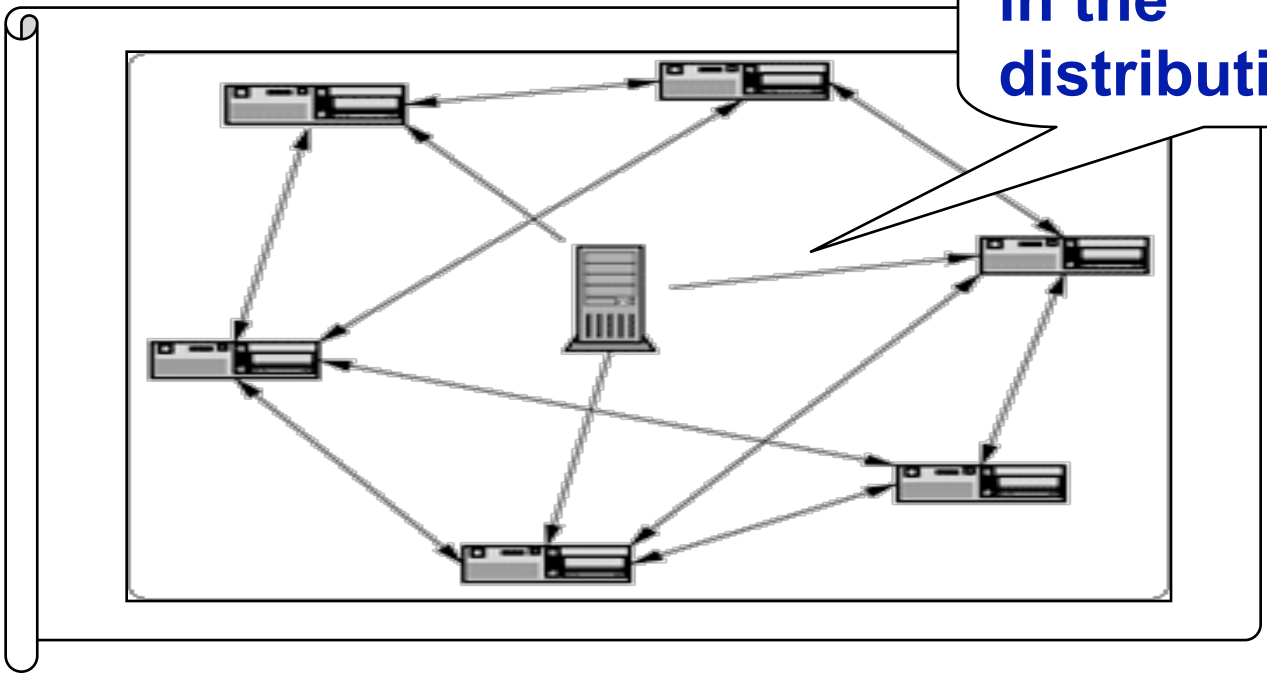


BitTorrent Solution

Users cooperate in the distribution



Publishing Content

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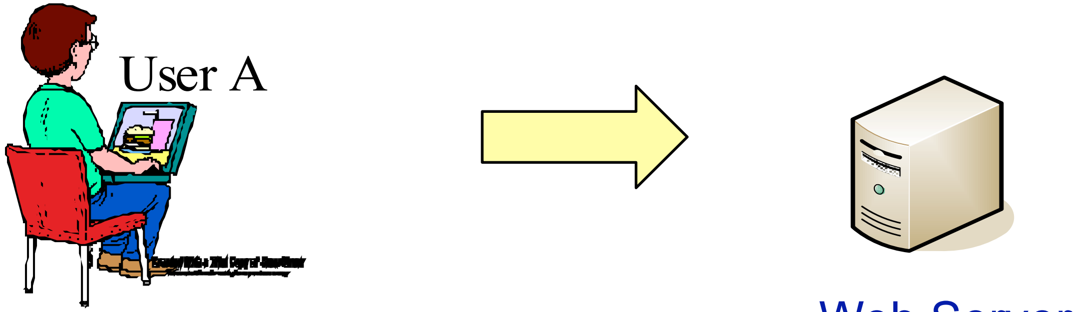

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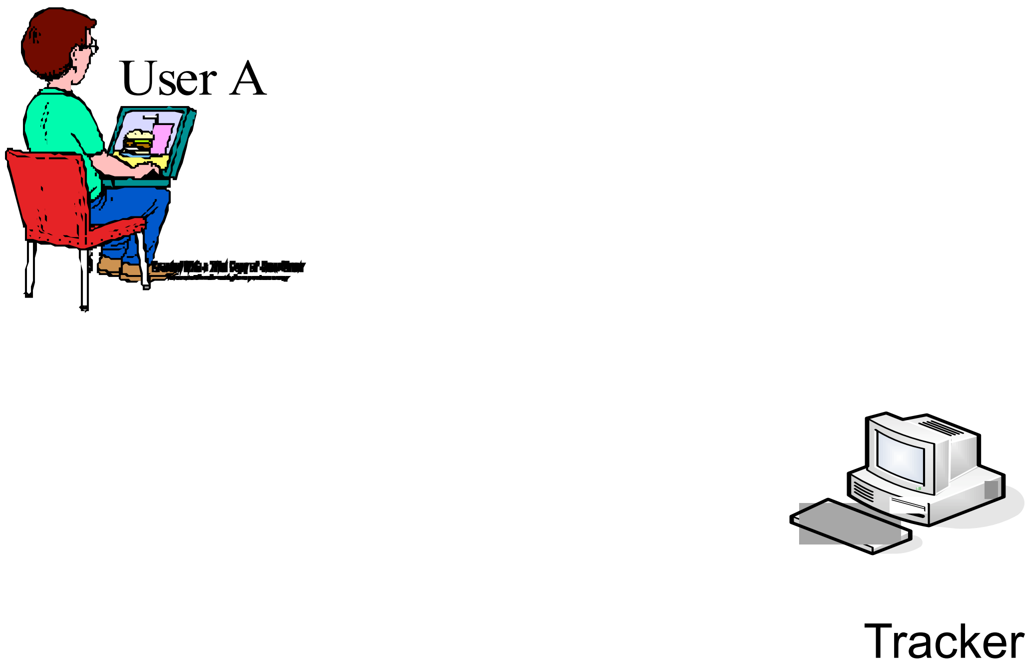

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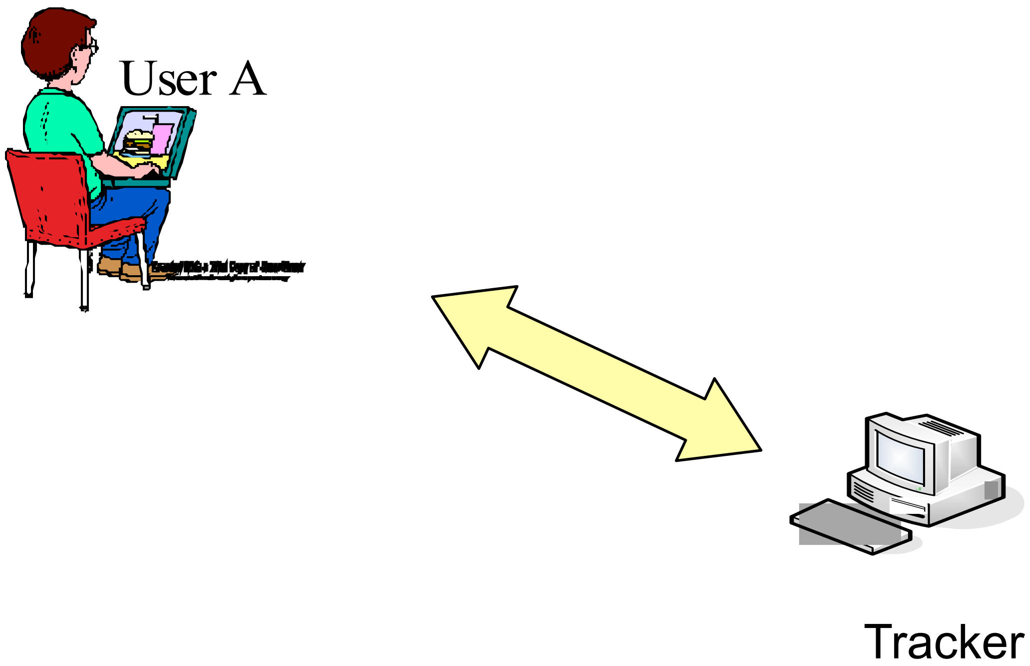

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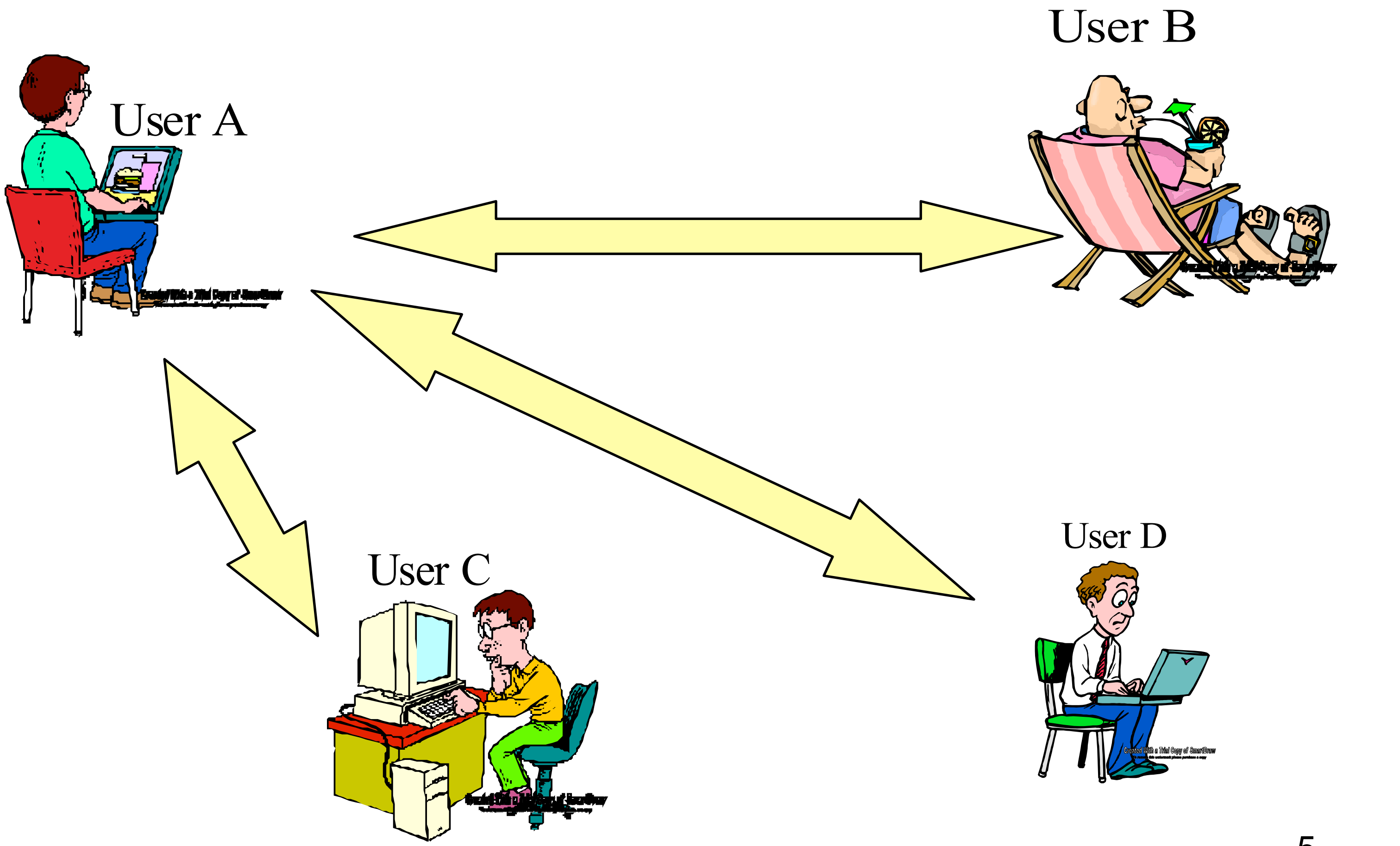

Motivation

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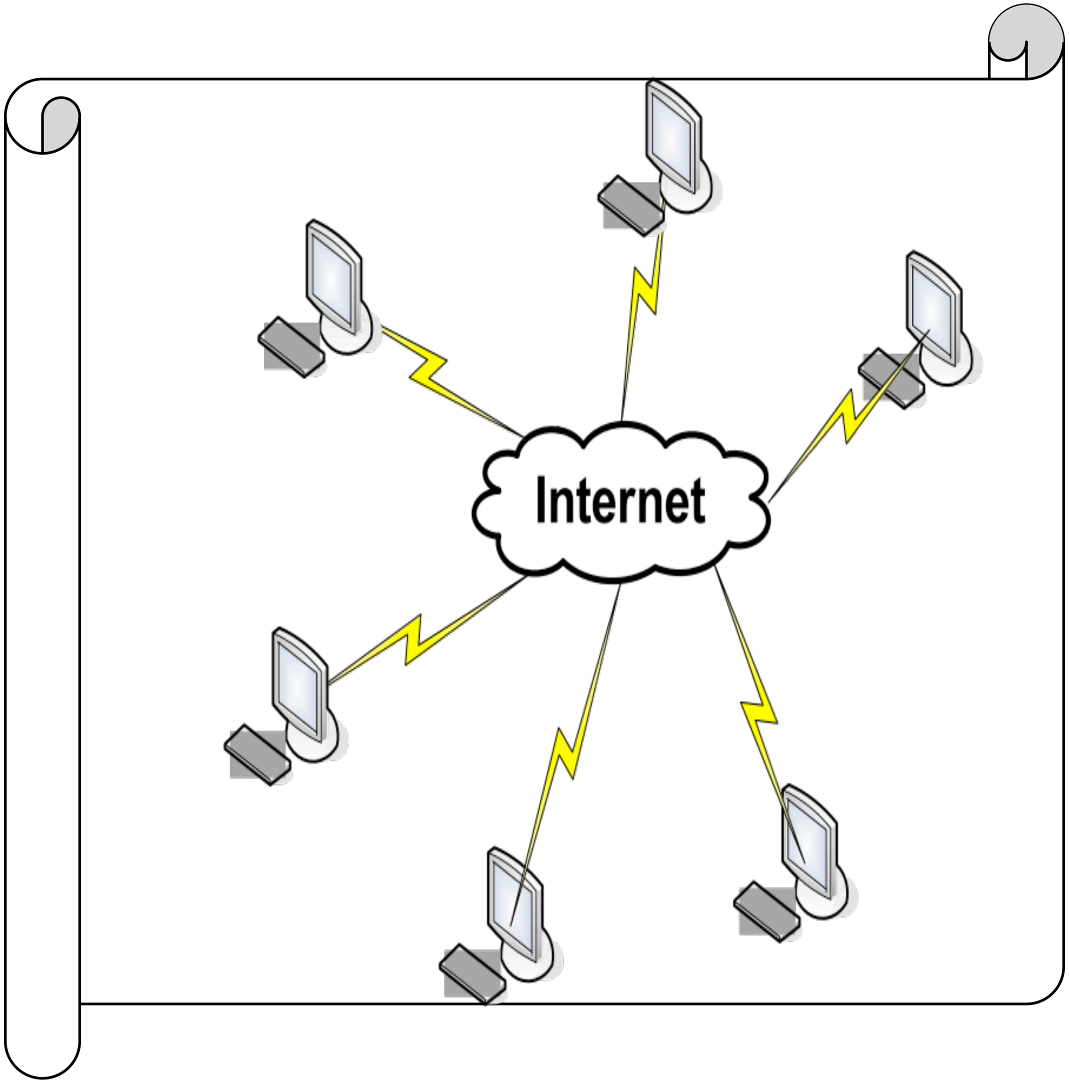
  - **Why is it good?**
  - **Can we do better?**
- **Contribution:**
  - **Analytical metrics**
  - **Insights for Protocol-designers**

Related Work

- **X. Yang and et al, Infocom 2004:**
  - Simple Markov Model
  - Numerical calculation needed
- **L. Massoulie and et al, Sigmetrics 2004**
  - Detailed Markov Model
- **D. Qiu and et al, Sigcomm 2004**
  - Simple Fluid Model
  - Some analytical results are obtained

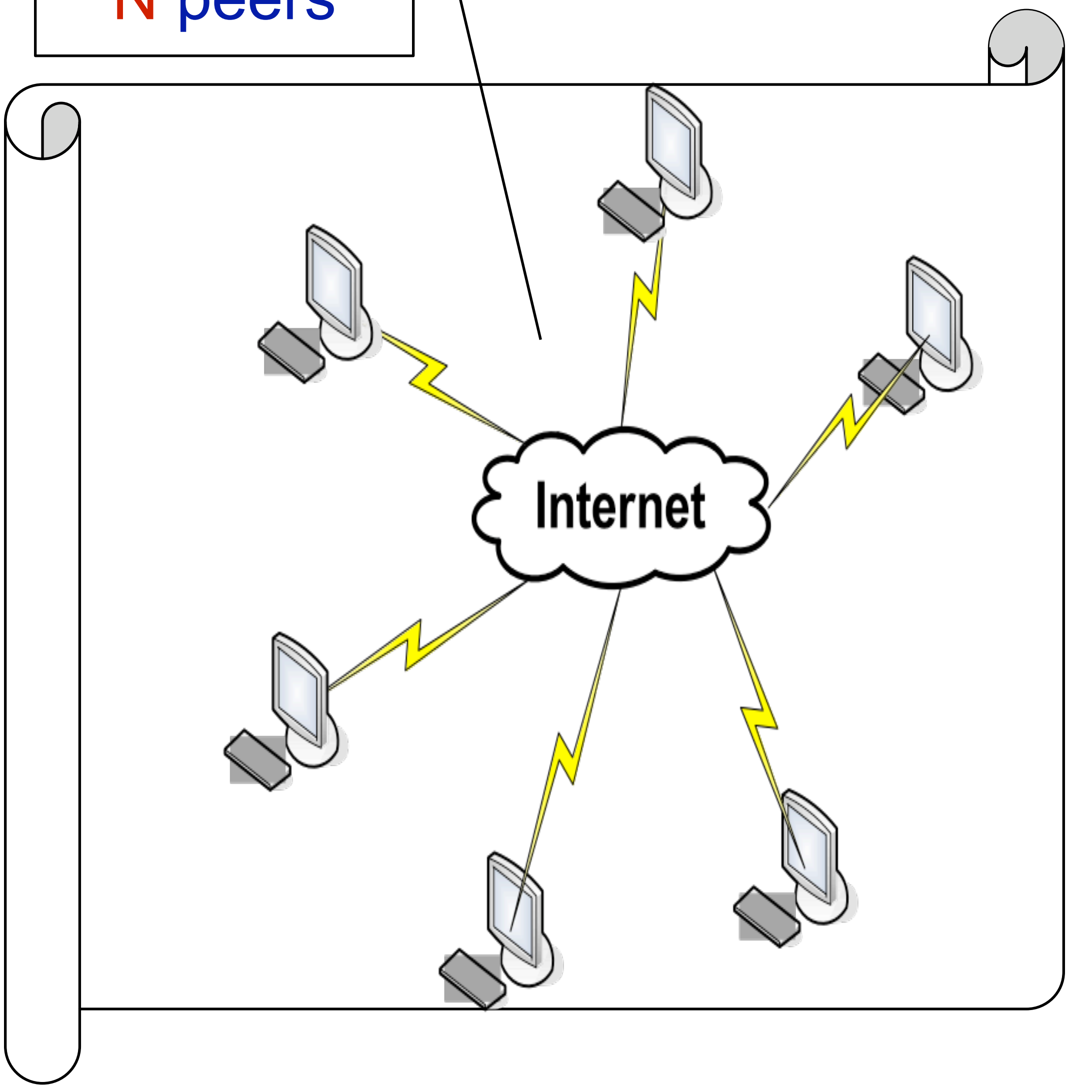
BitTorrent Model

BitTorrent Model



BitTorrent Model

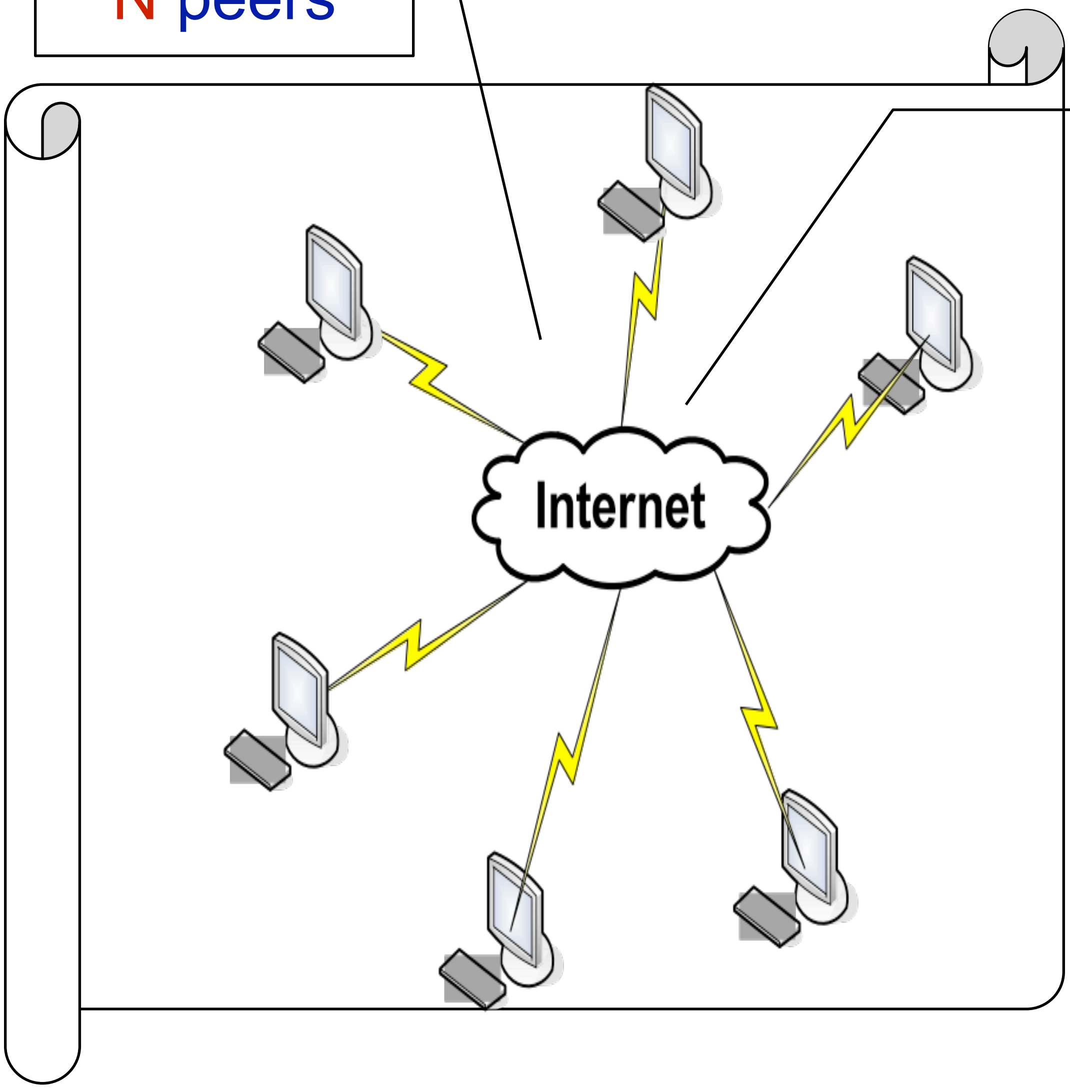
N peers



BitTorrent Model

N peers

The File is divided into M chunks



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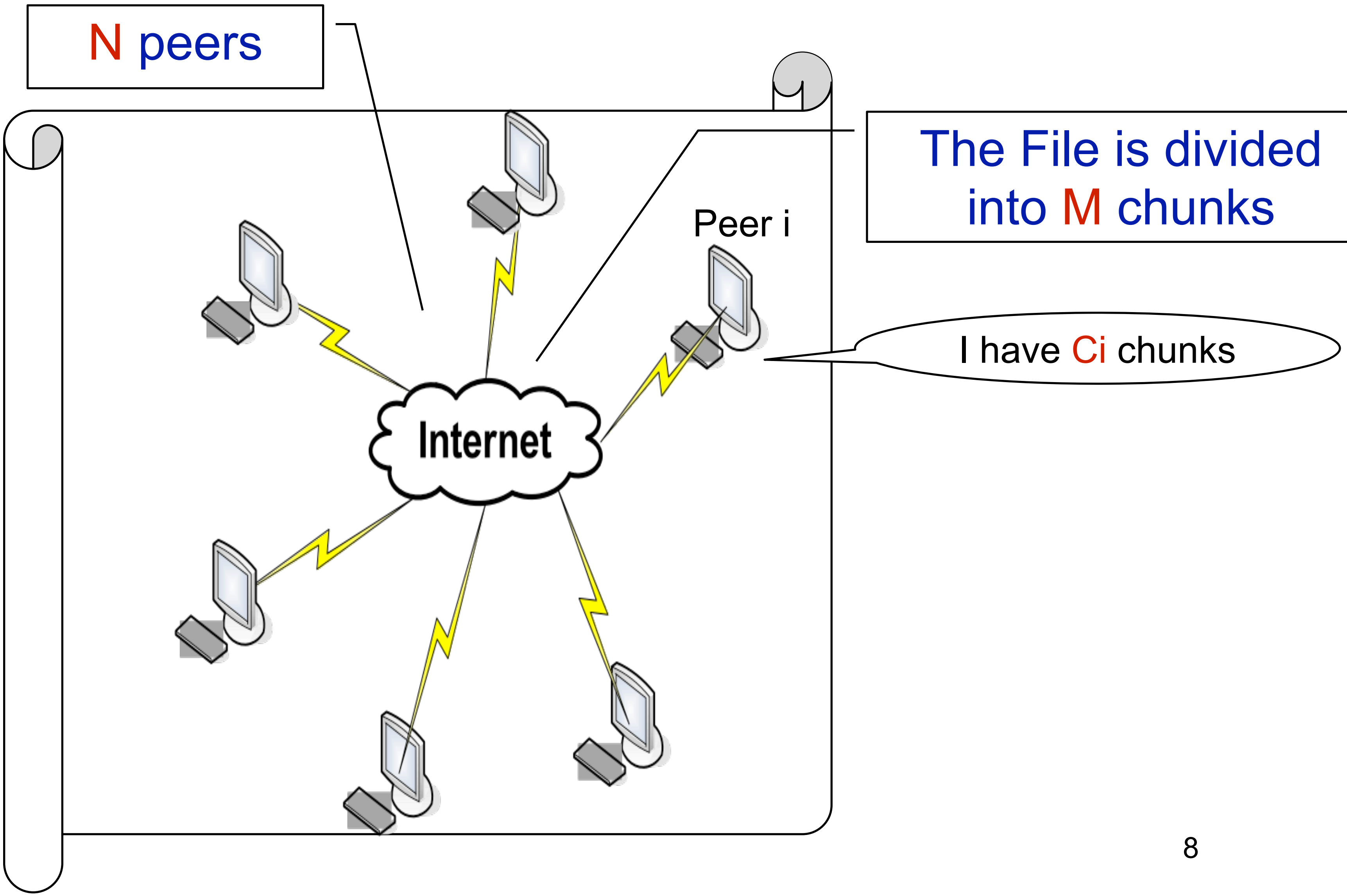

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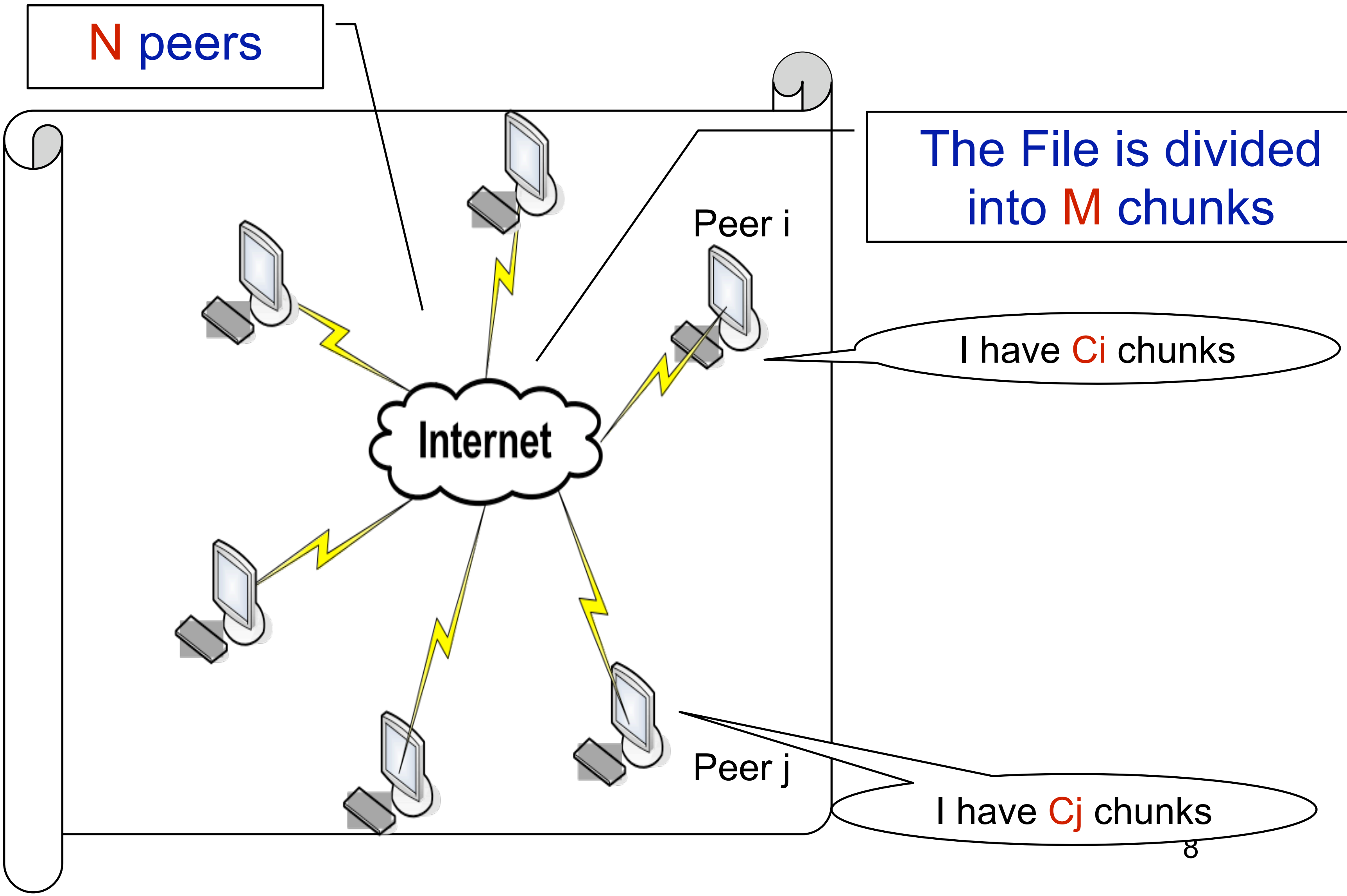

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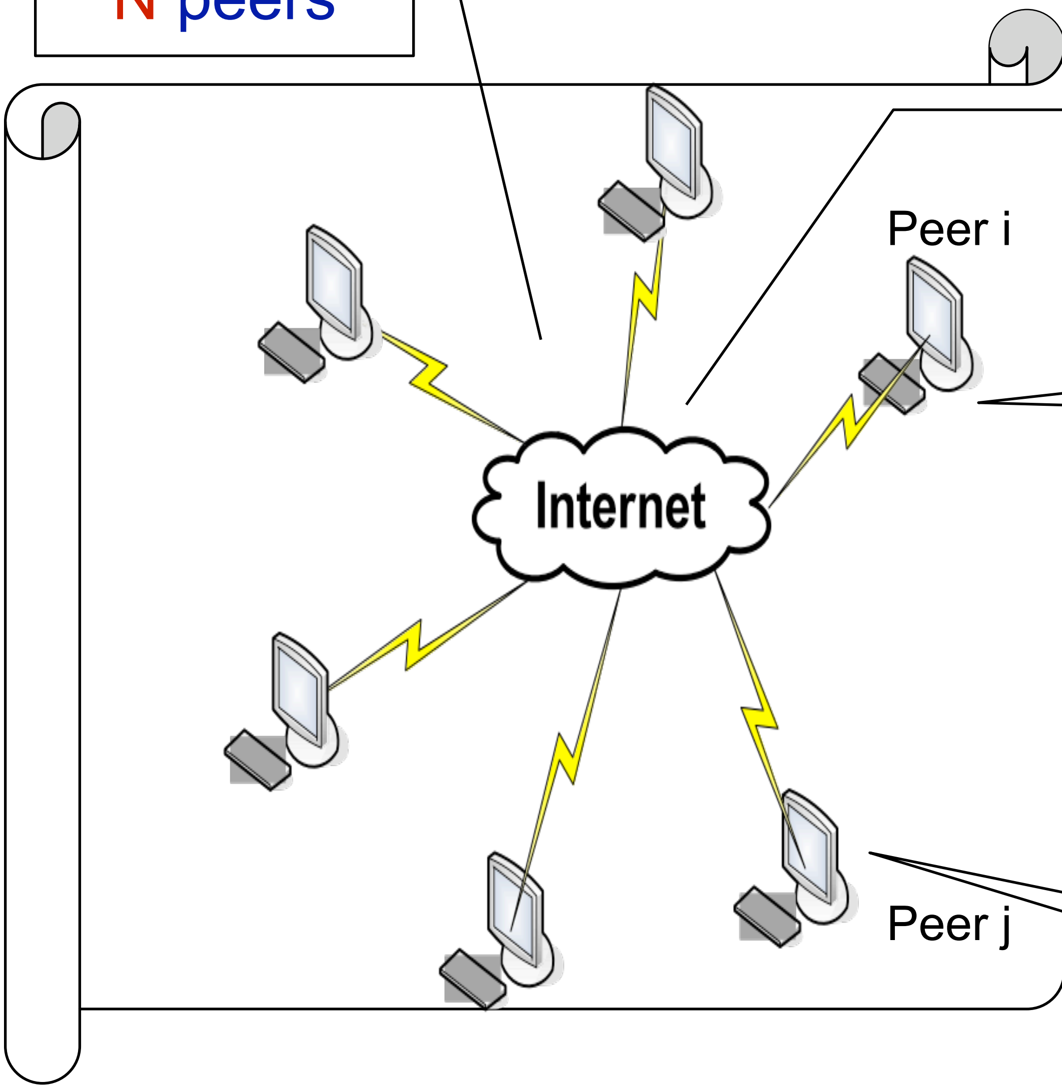

The File is divided into M chunks

I have Ci chunks

$$P_{i,j} = 1 \quad \text{When } c_i < c_j$$

$$P_{i,j} = \quad 1 - \frac{\binom{c_i}{c_j}}{\binom{M}{c_j}}$$

When $c_i \geq c_j$

I have Cj chunks

Simplified Peer States

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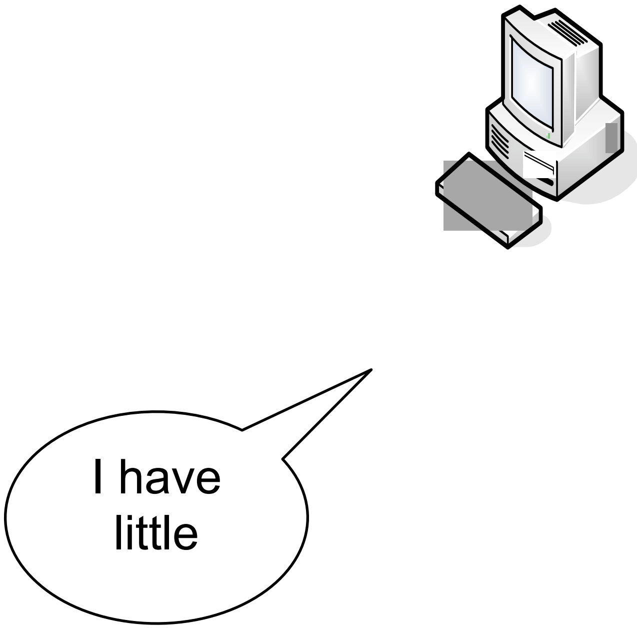

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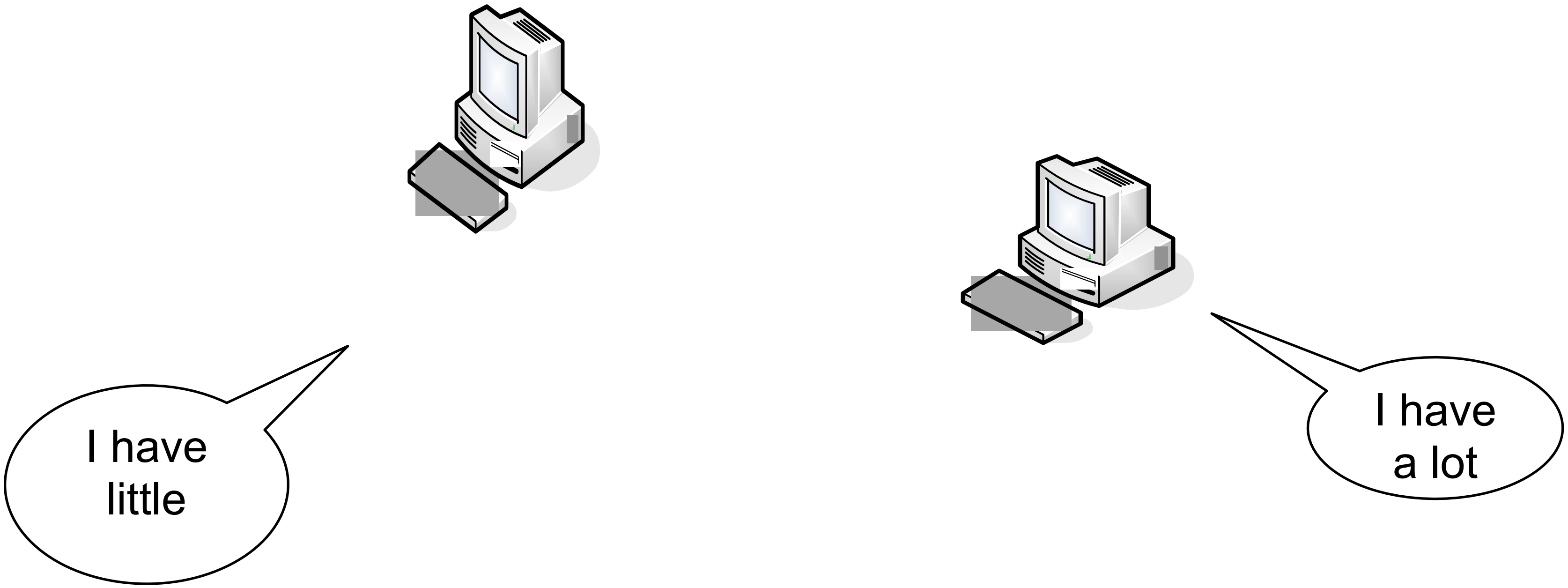

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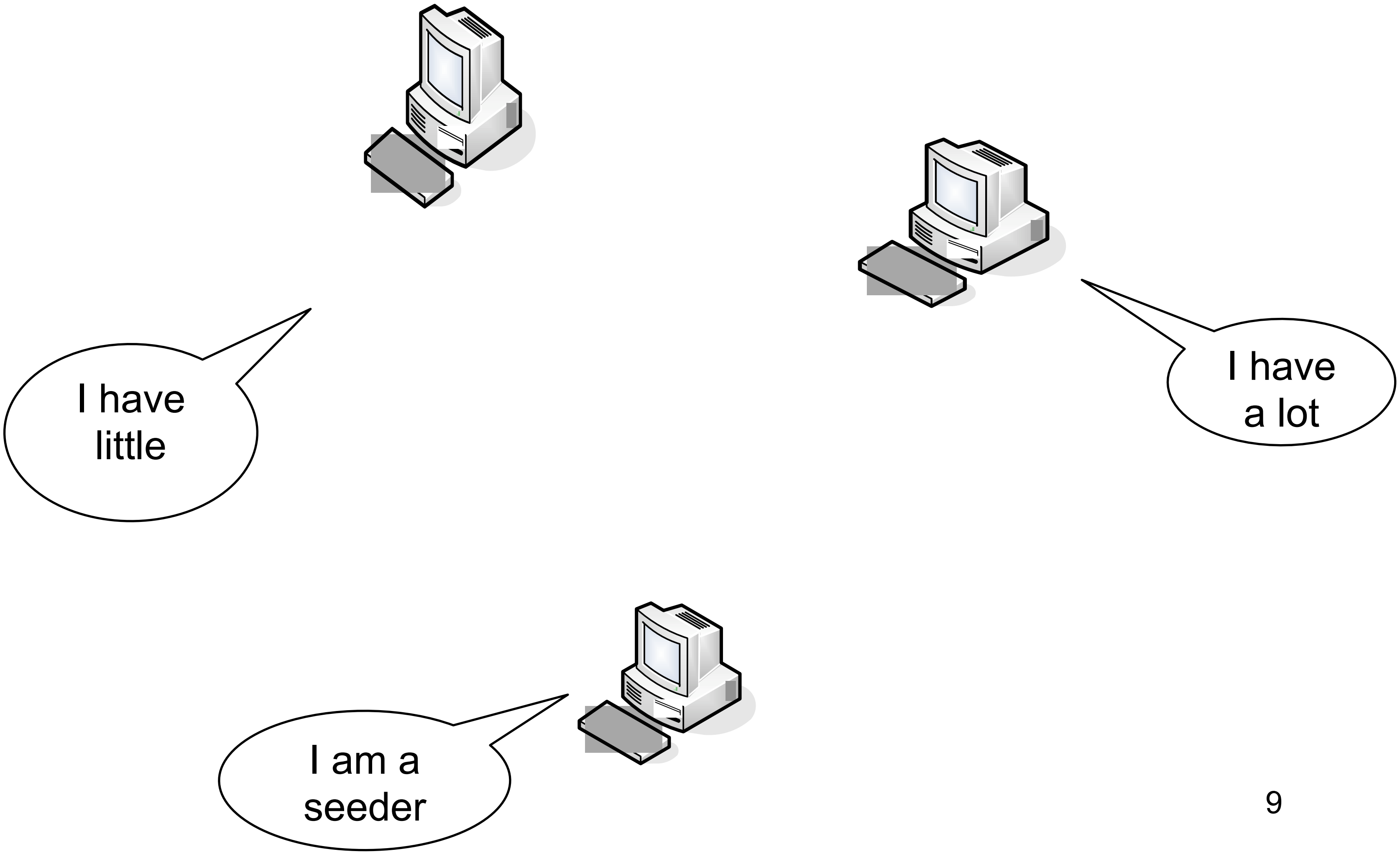

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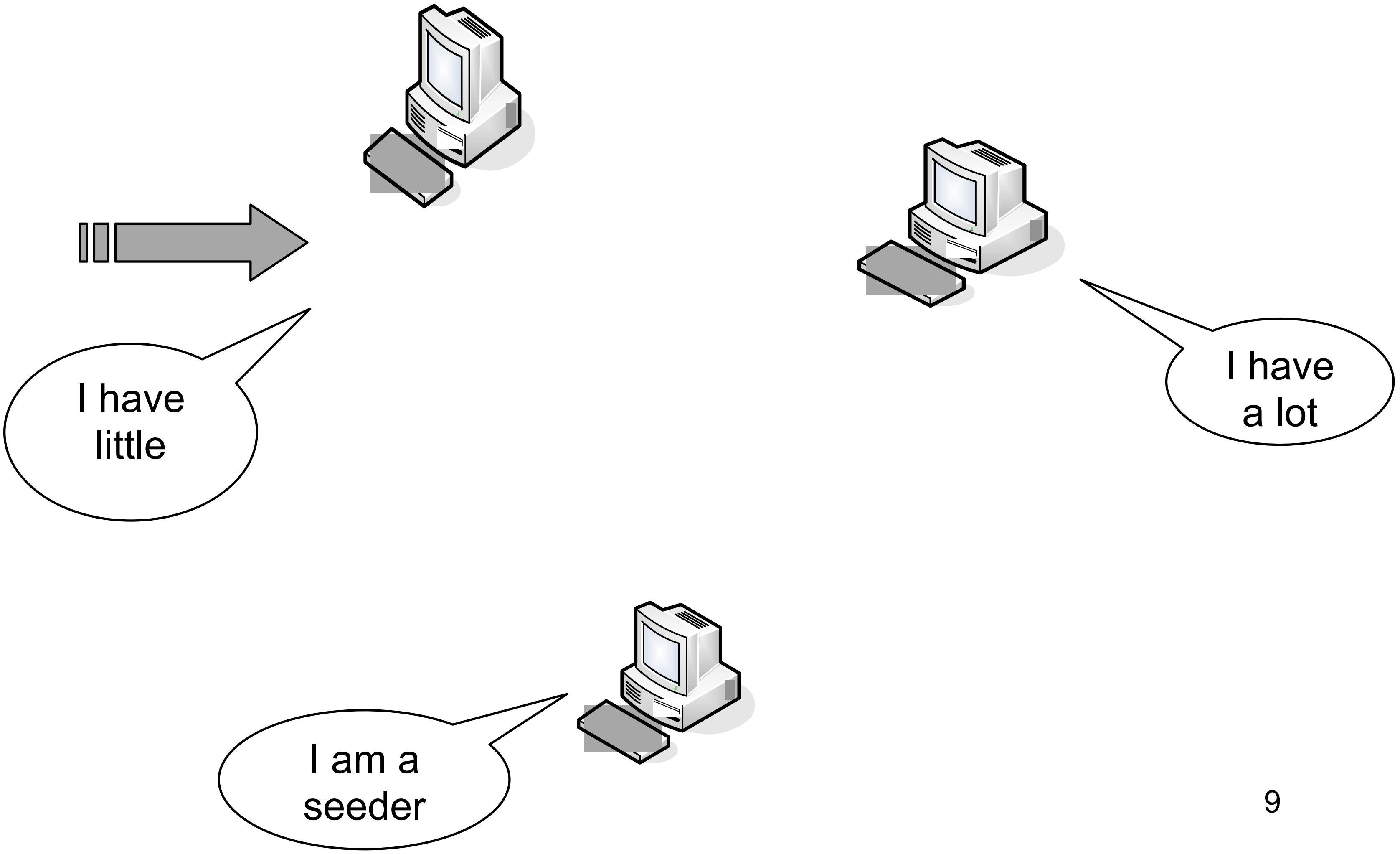

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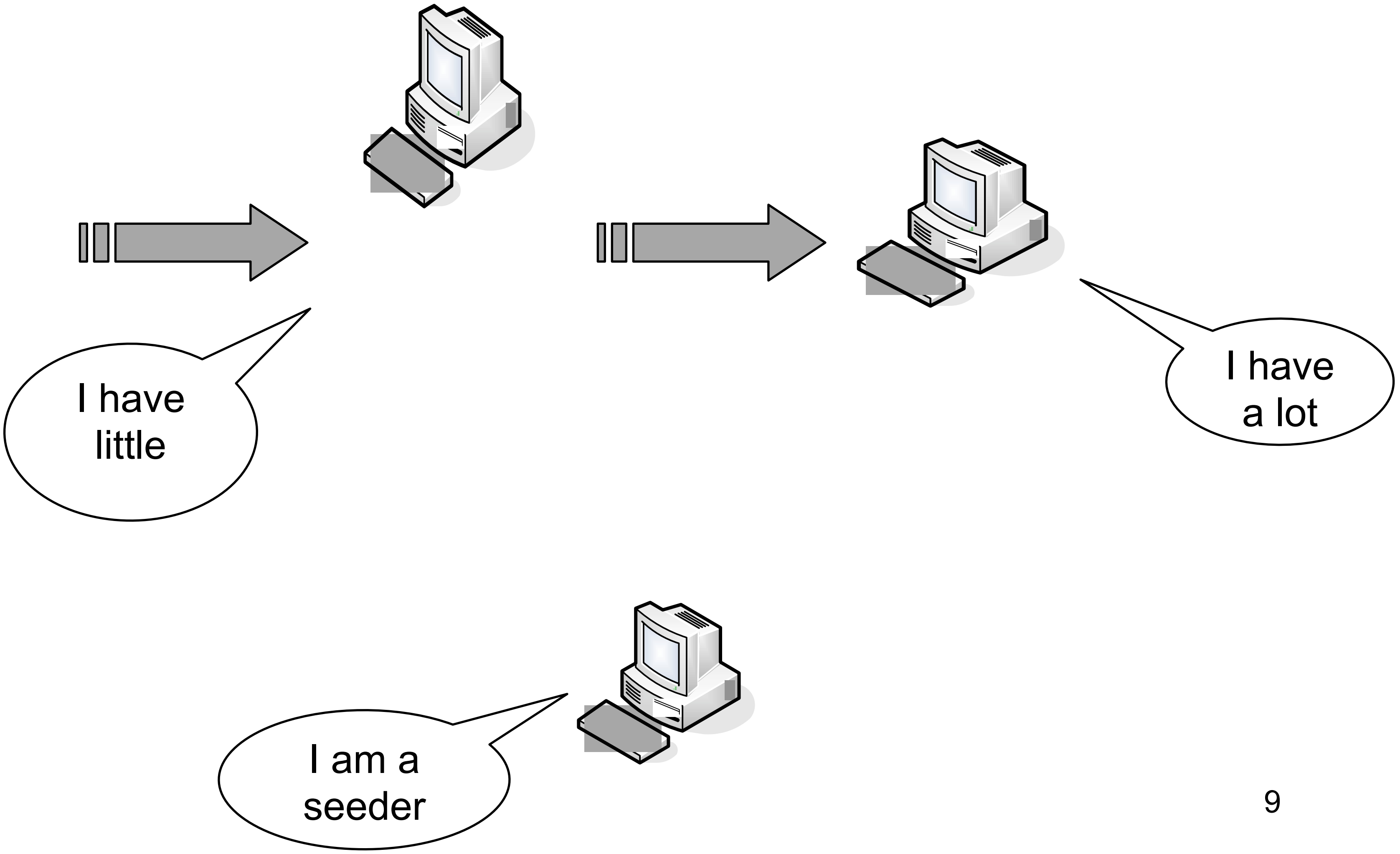

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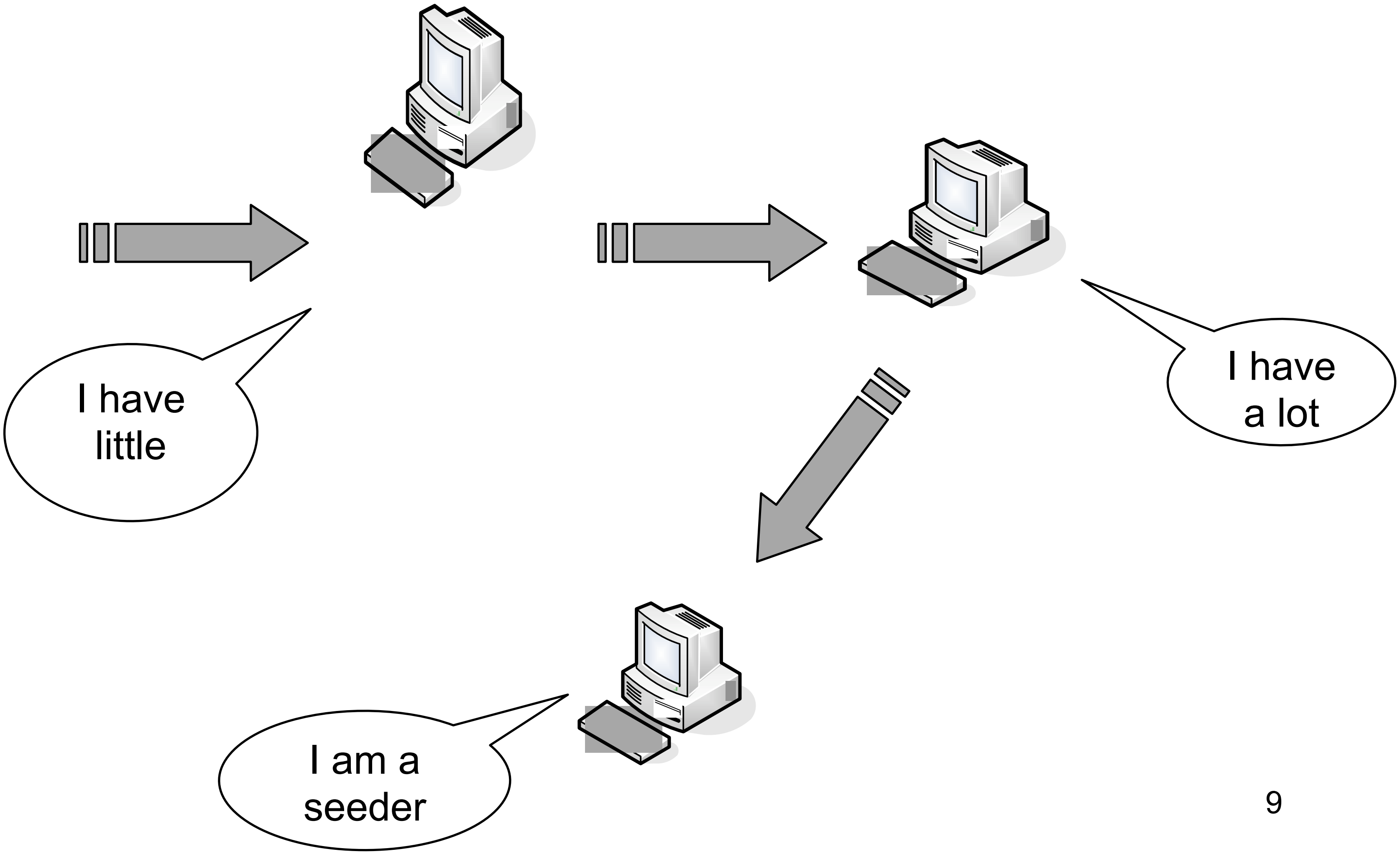

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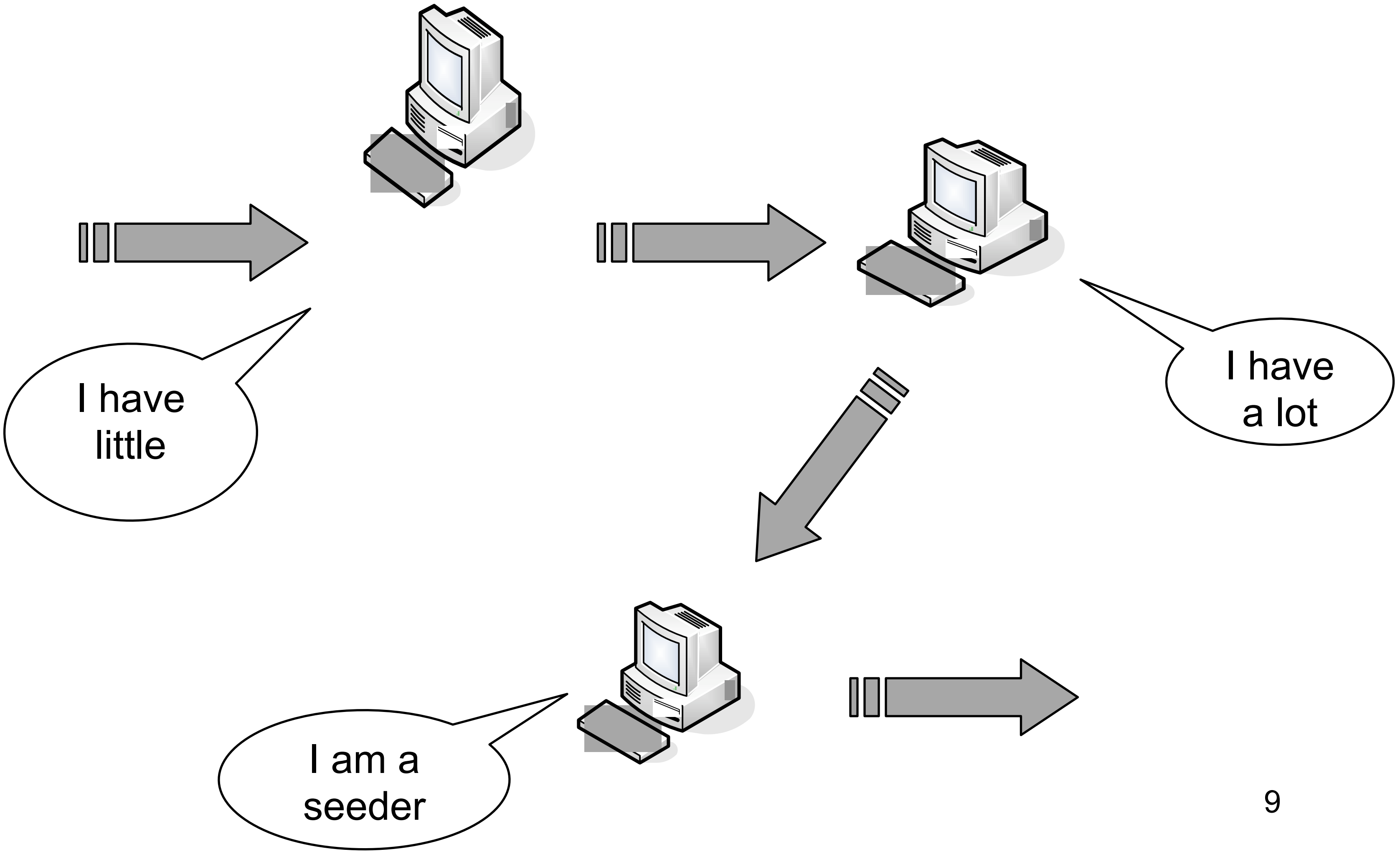

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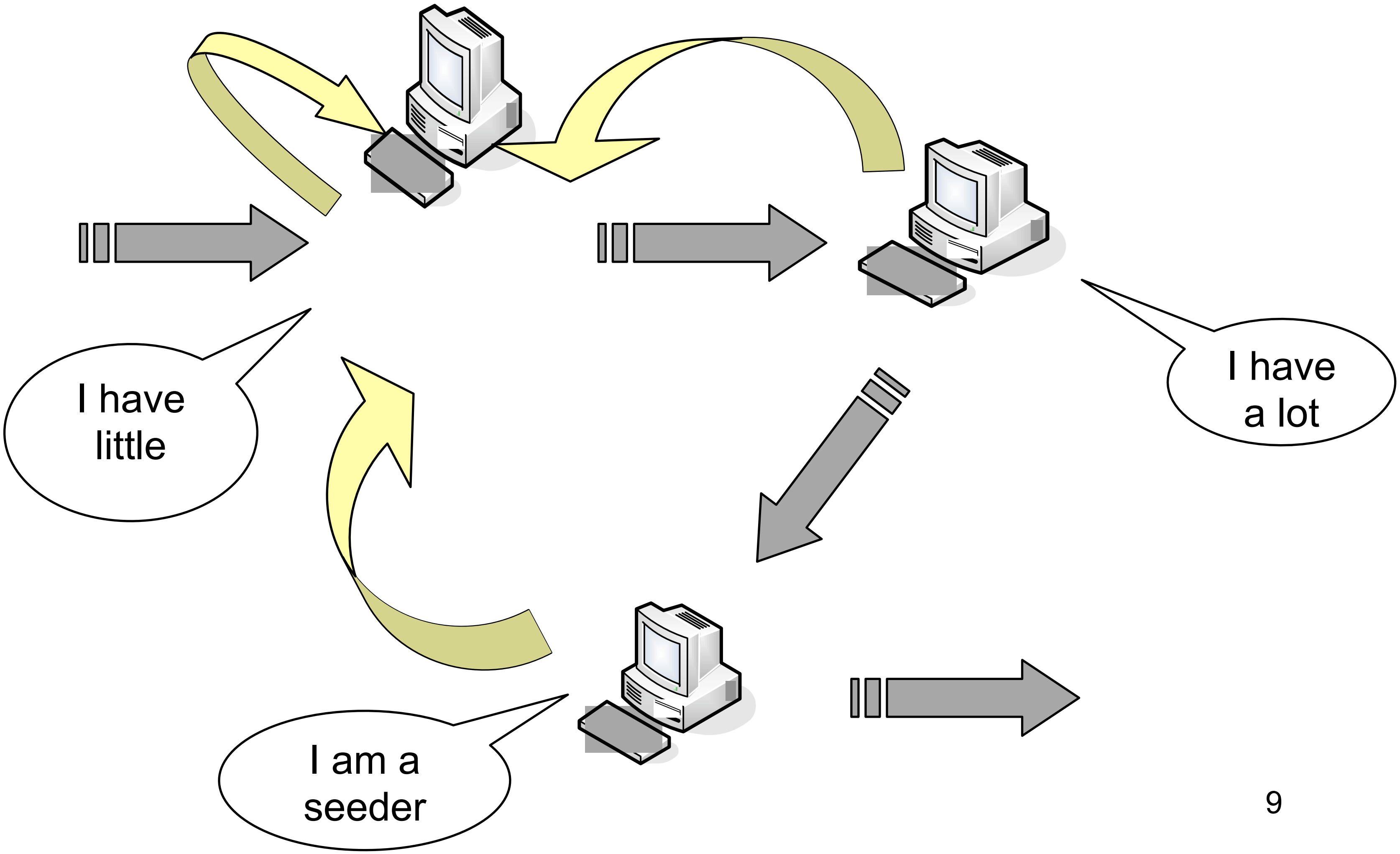

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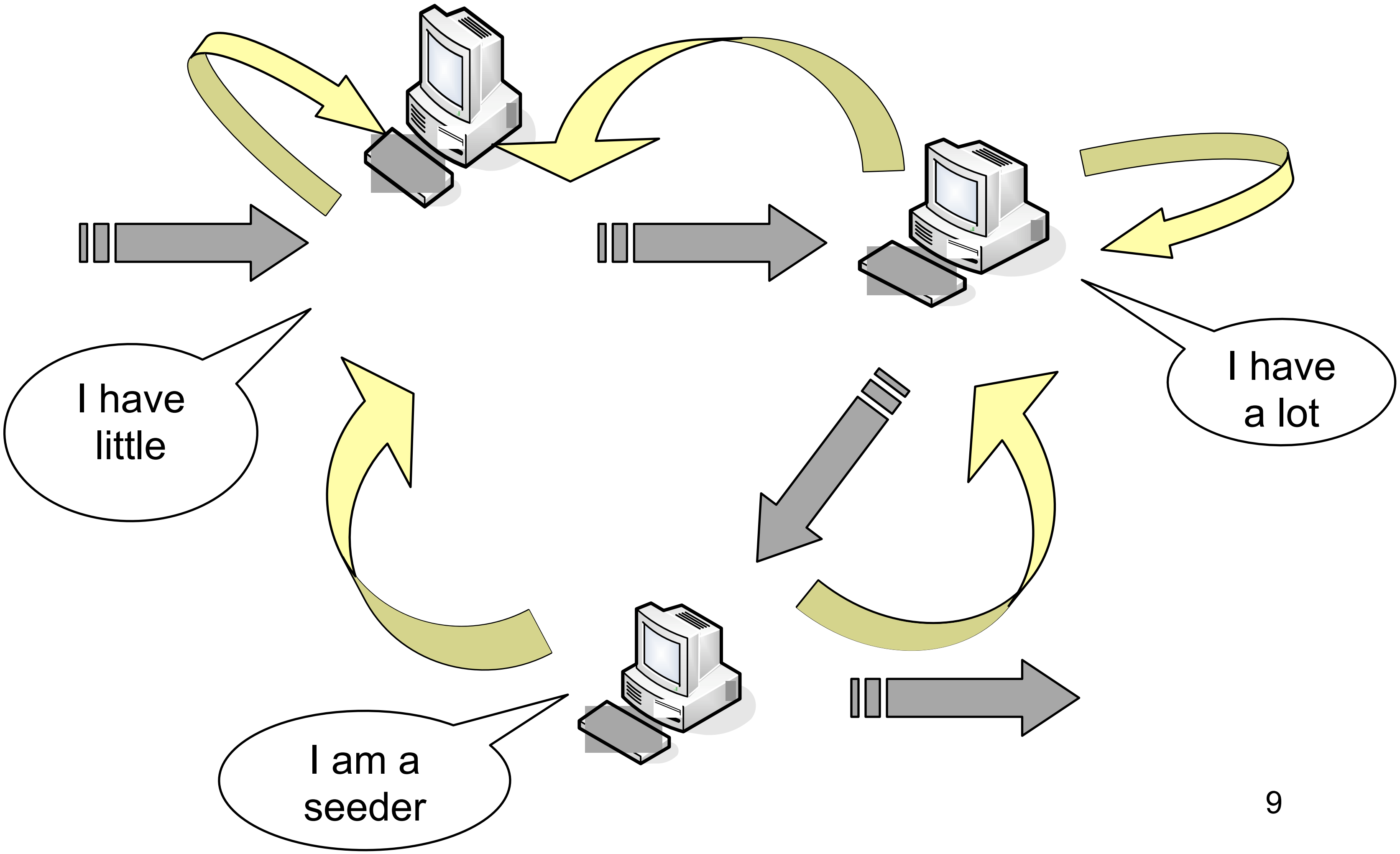

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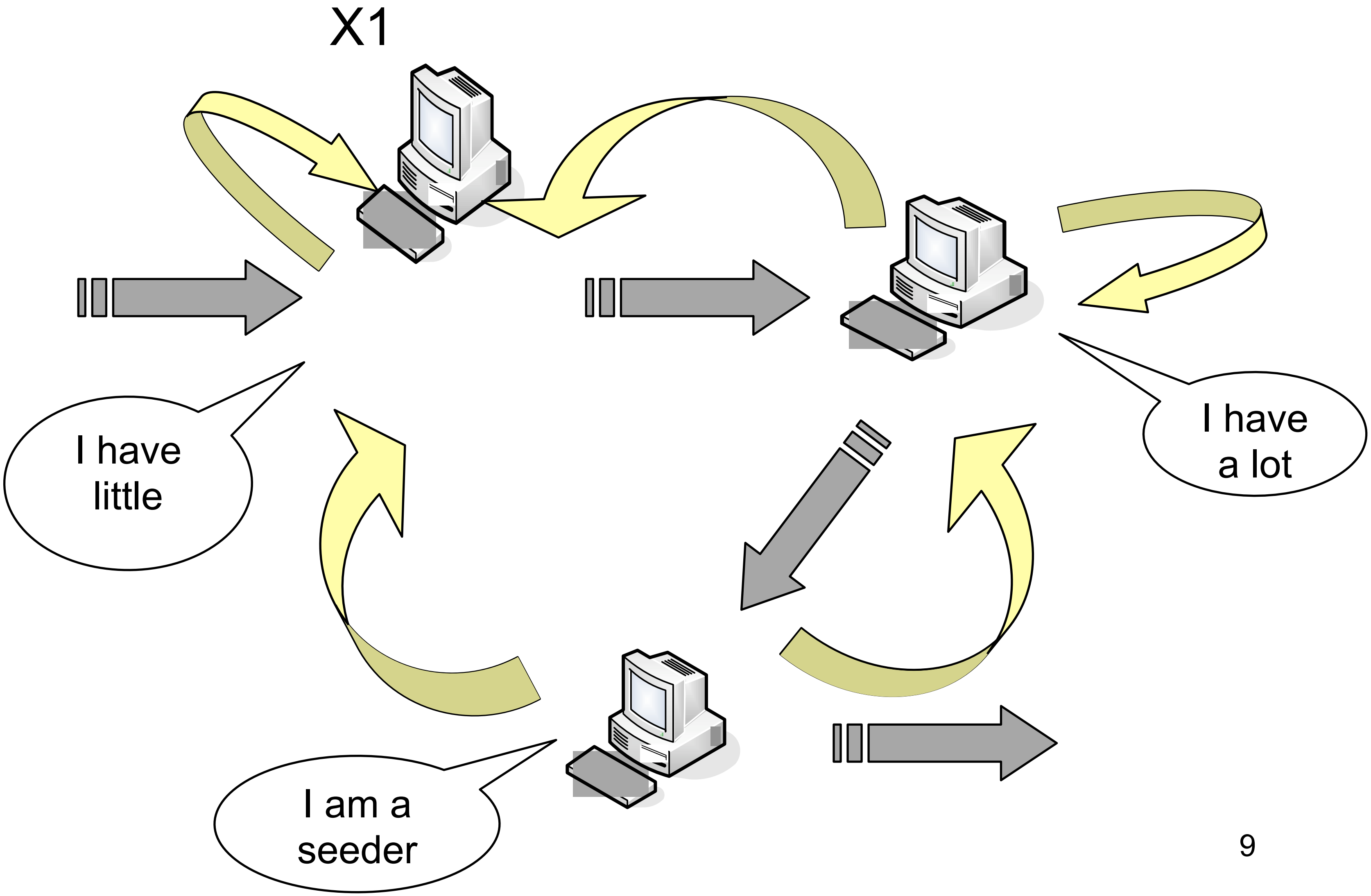

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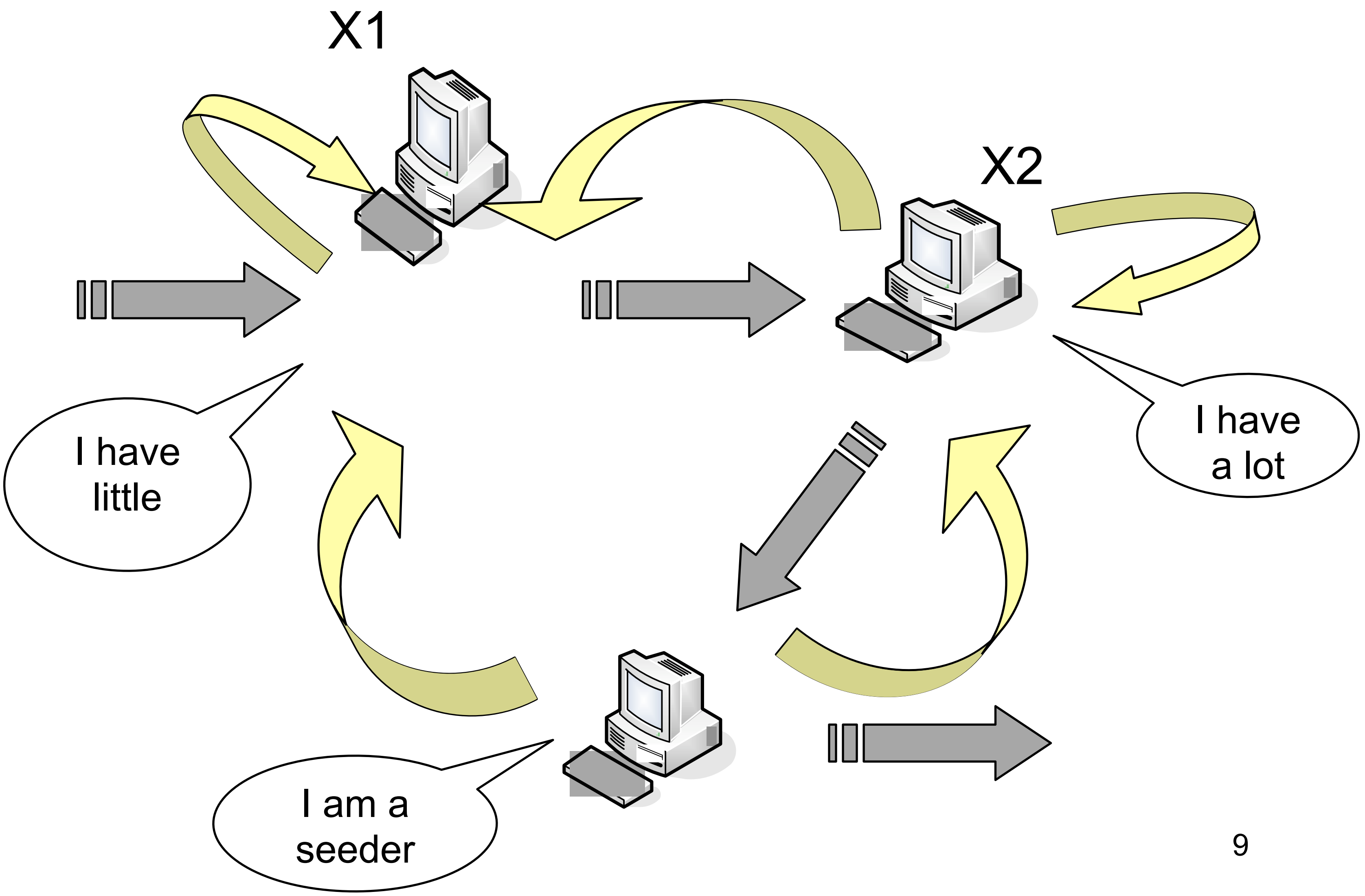

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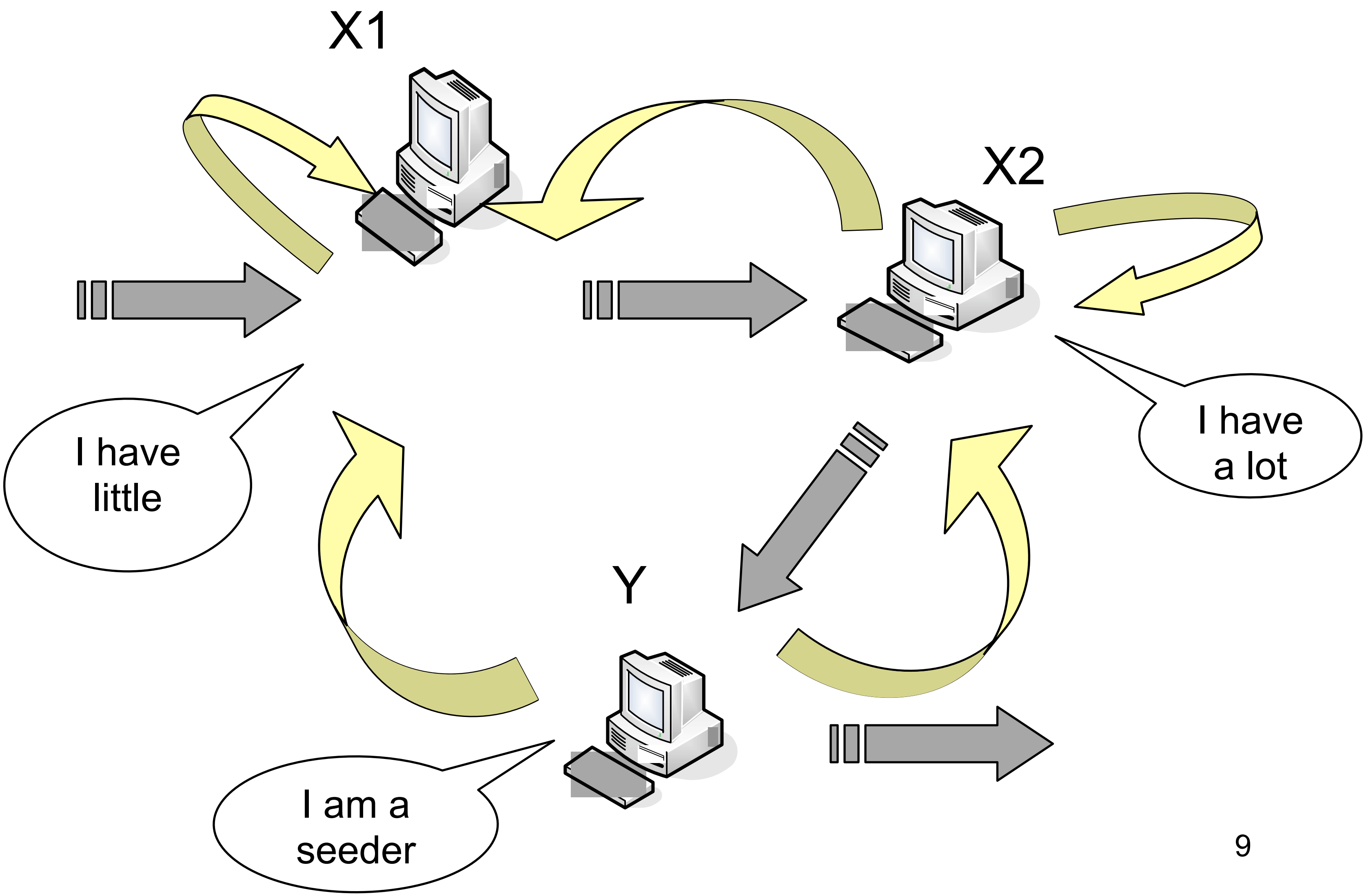

Dynamics

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Transfer Rate from State 2 to State 3

$$R_1 = X1 \times \min\{\mu\rho(X1 + X2 + Y), B/2\} / (sM/2)$$

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Downloading rate per connection

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Bandwidth Constraint

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Chunk Size

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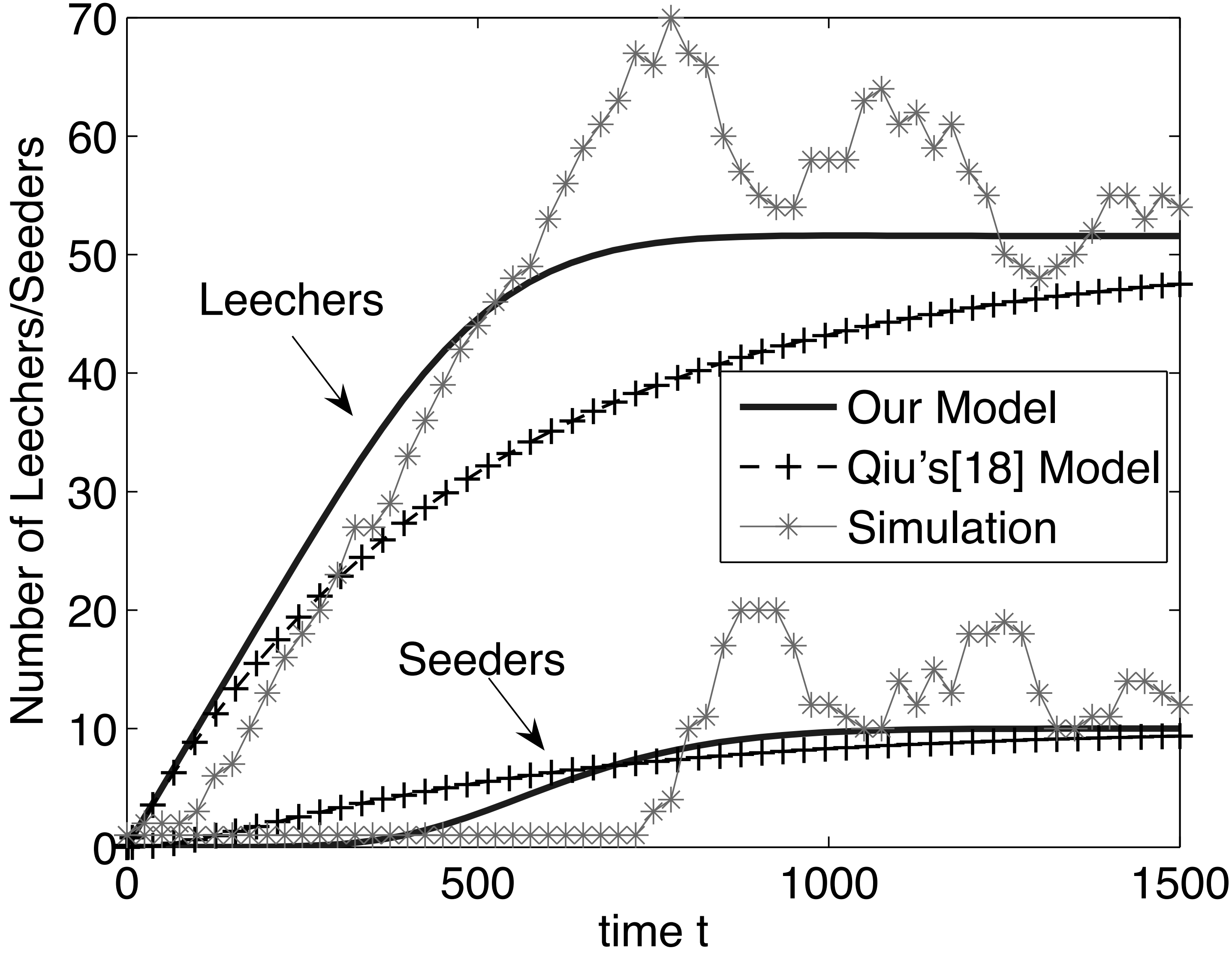

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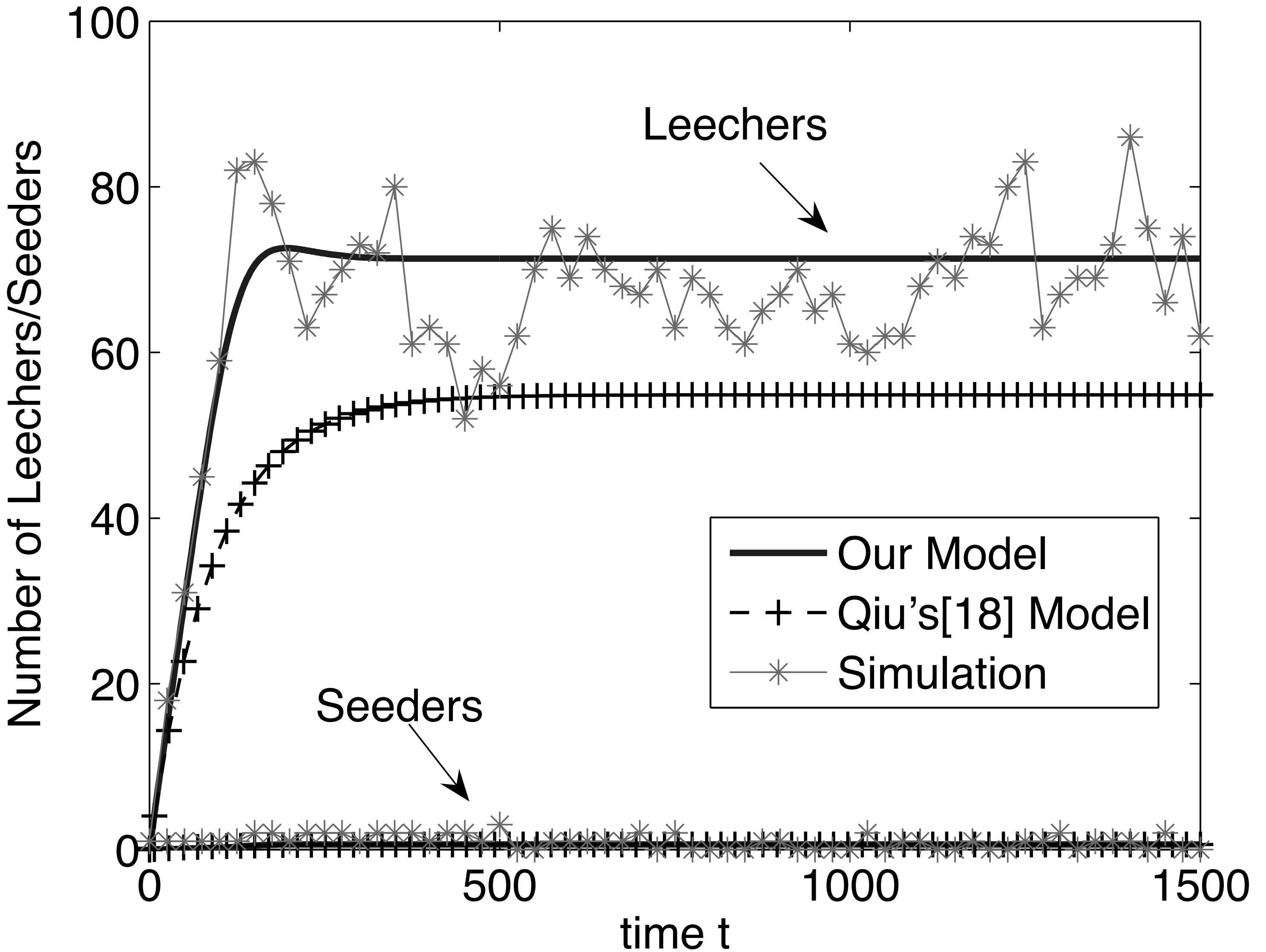

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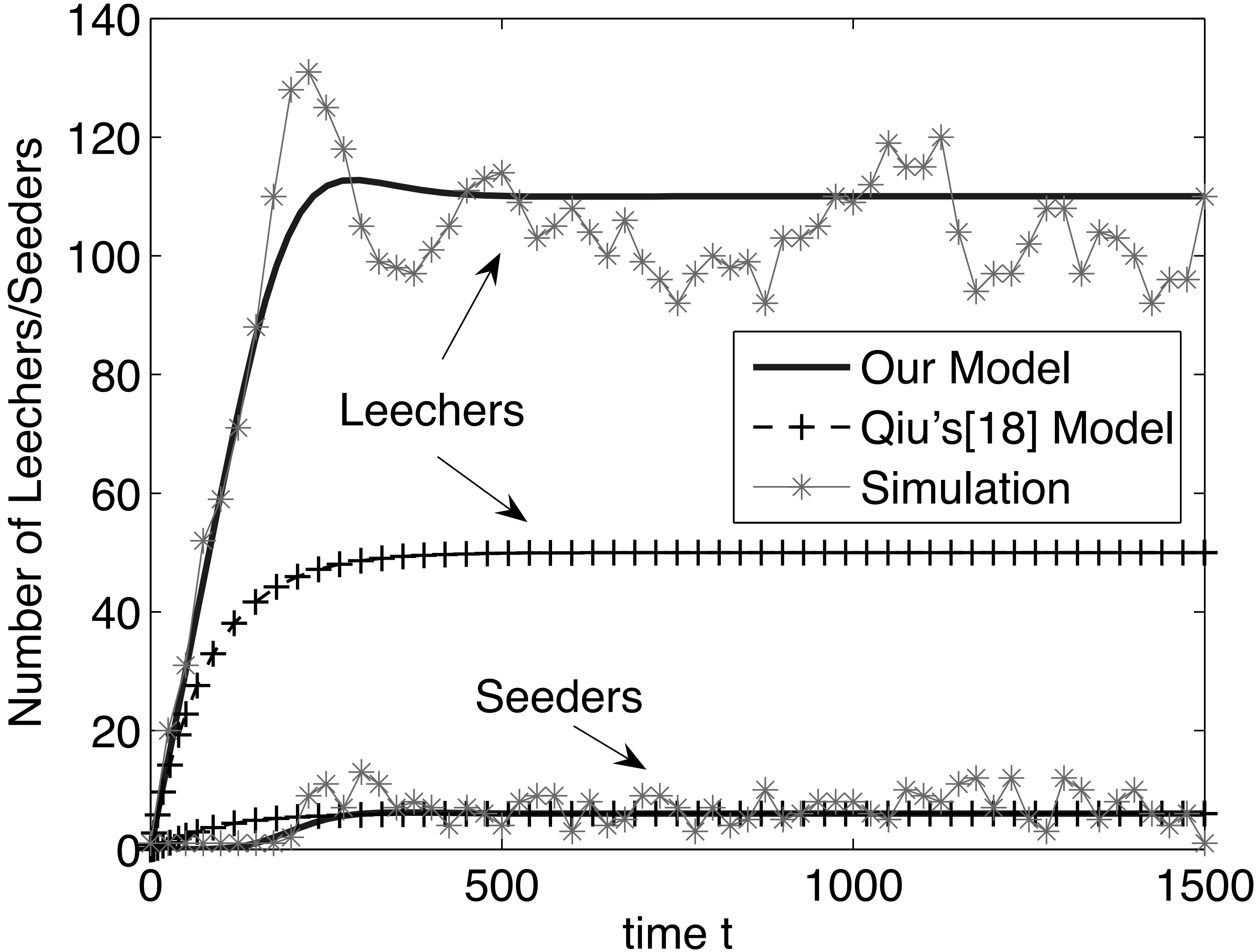

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Steady State

$$\bar{T}_d = \frac{\bar{X}_1 + \bar{X}_2}{\lambda}$$

$$\bar{T}_p = \begin{cases} O(\bar{N}^2) & \textit{Case 1,} \\ O(\bar{N}) & \textit{Case 2 or 3.} \end{cases}$$

Steady State

- **By Little's Law, average downloading time in the steady state is derived by:**

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- The system throughput in steady state is derived by:

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Insights: Scalability

- **The system shows good scalability:**
  - **Tp=O(N^2) in Case 1**
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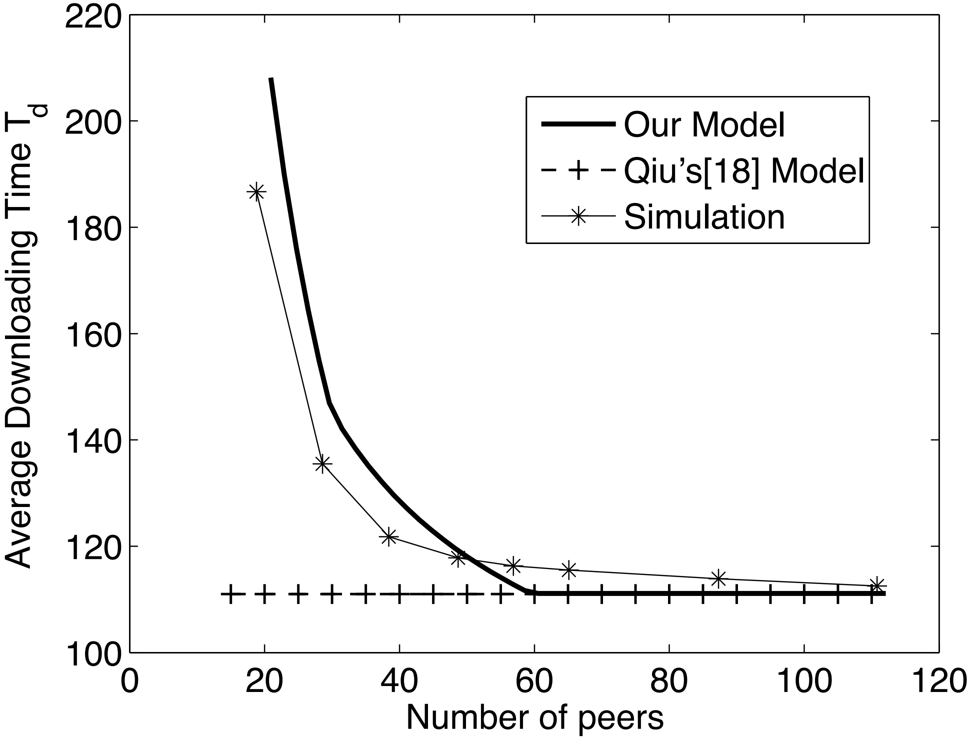

(a) $\bar{T}_d$ as the function of N

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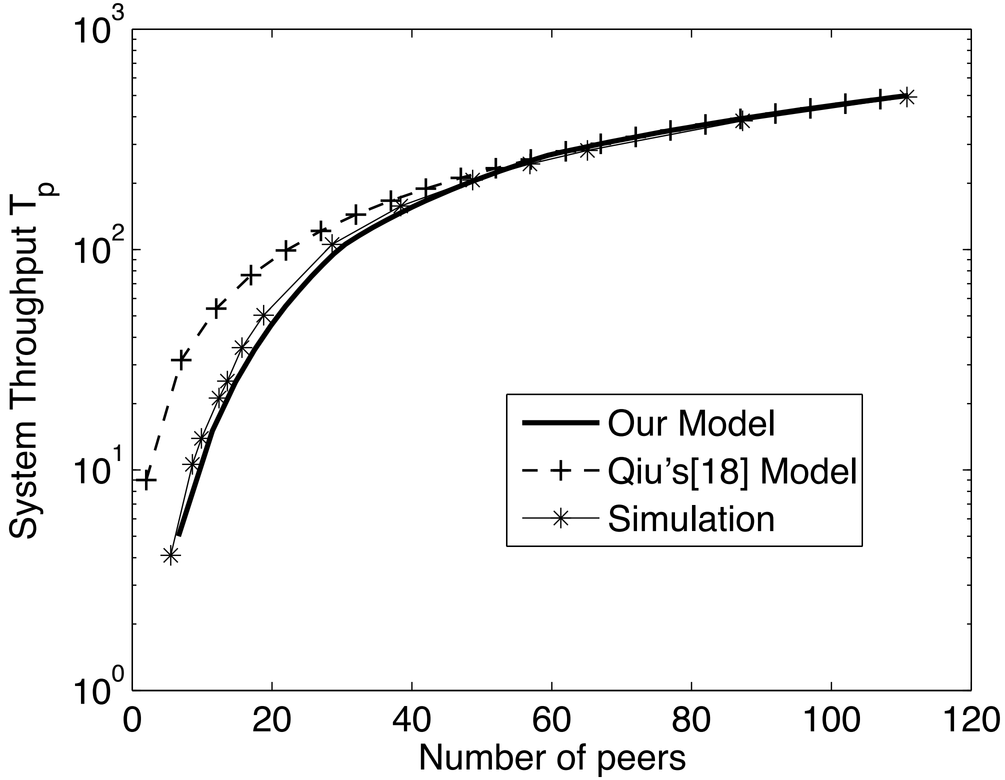

(b) $\bar{T}_p$ as the function of N

Insights: Scalability

- **The system shows good scalability:**
  - **Tp=O(N^2) in Case 1**
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Insights: Popularity

- **The arrival rate λ represents the popularity of the served file.**

$$\frac{\partial \bar{T}_d}{\partial \lambda} = \begin{cases} -\frac{1+\sqrt{5}}{4\sqrt{\alpha}}\lambda^{-3/2} & \text{Case 1,} \\ -\frac{1}{2\sqrt{\alpha}}\lambda^{-3/2} & \text{Case 2,} \\ 0 & \text{Case 3.} \end{cases}$$

  - More popular the file is, less downloading time, in Case 1 and 2.
  - The downloading time keeps the same in Case 3.

Insights: Seeding

- Let $T_s = 1/\gamma$ be the average seeding time
- Increase seeding time T_s:
  - less downloading time T_d in case 1 and 2;
  - same downloading time T_d in case 3.
- Extreme situation: T_s=0:
  - Downloading time T_d won't be infinity

Insights: Topology

- The average degree of a peer in overlay:

$$\rho(N-1)$$

- This degree is affected by the list returned by tracker (30-60 by default)
- Larger ρ :
  - reduce T_d in case 1 and 2
  - won't help in case 3, only burden the network

Insights: Bandwidth

- Larger B:
  - reduce T_d in case 2 and 3
  - won't help in case 1

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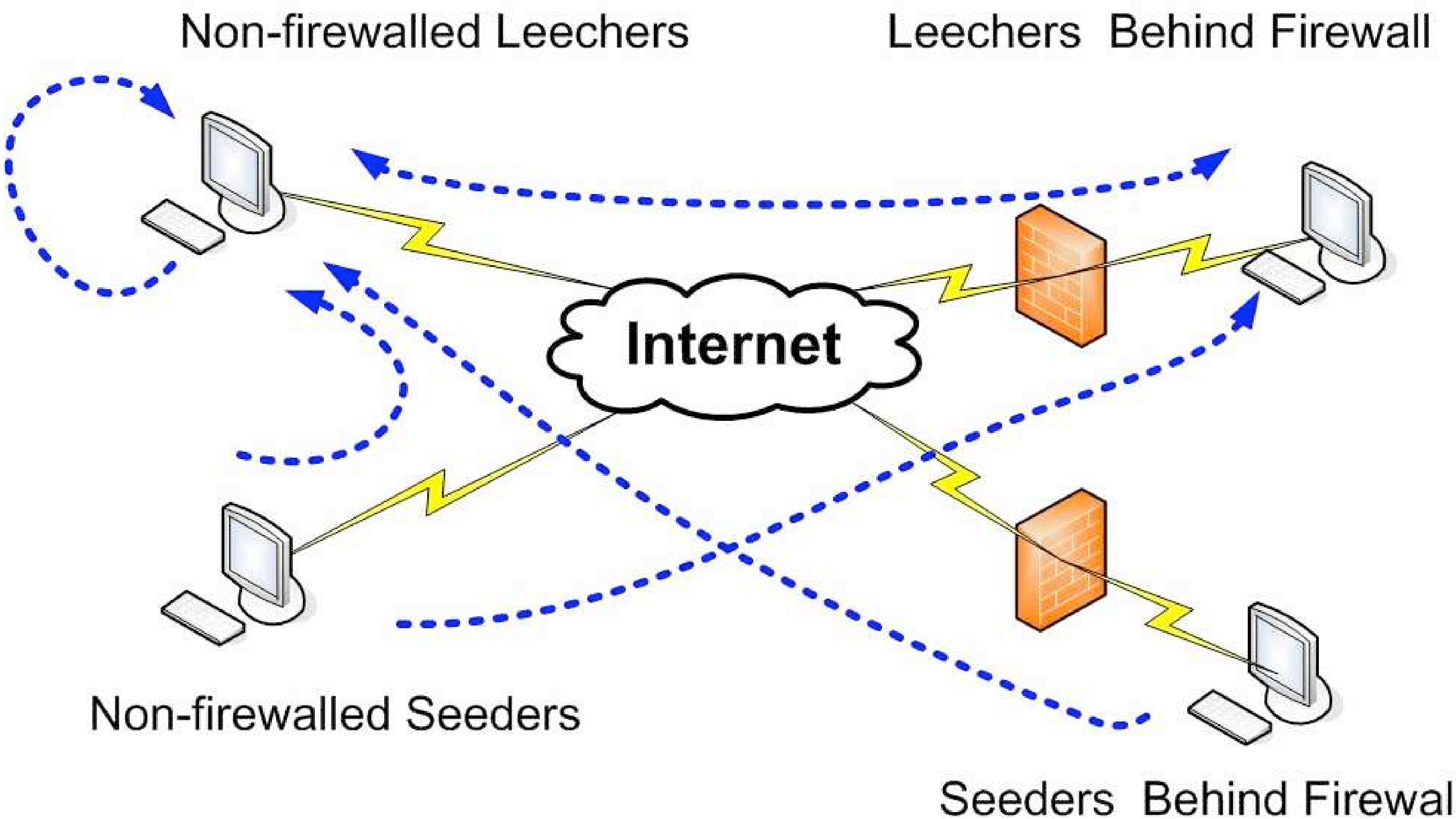
Impact of Firewall



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$$\dot{X}f = \lambda f - Xf \min\{\mu\rho Xp, B/2\}/sM$$

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Arrival Rate of non-firewalled peers

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Number of non-firewalled peers

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- Without non-firewalled peers, the peers behind firewalls can not finish downloading
- Non-firewalled peers perform better
- The performance gap is related to the arrival rate

This gap can be very large even the two arrival rates are very close

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File Availability

File Availability

- The probability that a peer can get ith chunk from its neighborhood:

File Availability

- The probability that a peer can get ith chunk from its neighborhood:

$$\gamma_i = 1 - \left(1 - \frac{h_i}{n}\right)^{\rho(n-1)} \approx 1 - e^{-\rho h_i}$$

File Availability

- The probability that a peer can get ith chunk from its neighbor

Number of copies of Chunk i

$$\gamma_i = 1 - \left(1 - \frac{h_i}{n}\right)^{\rho(n-1)} \approx 1 - e^{-\rho h_i}$$

File Availability

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- The probability that a peer can finish its download:

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Average number of neighbors

- The probability that a peer can finish its download:

$$\Theta = \prod_{i=1}^{M} \gamma_i$$

Optimal Chunk Distribution

$$Max \quad \Theta$$

$$s.t. \quad \sum_{i=1}^{M} h_i \leq C$$

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- The optimal solution:
  - h1=h2=…hM=C/M

Optimal Chunk Distribution

- Maximize the probability of downloading all chunks:

$$Max \quad \Theta$$

Chunks should be distributed as equally as possible

$$\sum_{i=1}$$

- The optimal solution:
  - h1=h2=…hM=C/M

Define the Measurement

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only increase hi when chunk i has the fewest copies

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only increase hi when chunk i has the fewest copies

Use Rarest First Policy

Synchronization problem occurs in high connectivity system

Where you are

- Introduction
- Modeling the Performance
- Extension on heterogeneous Network
- Modeling the Availability
- Availability Improvement Algorithms
- Conclusion

File Enhancement Algorithm

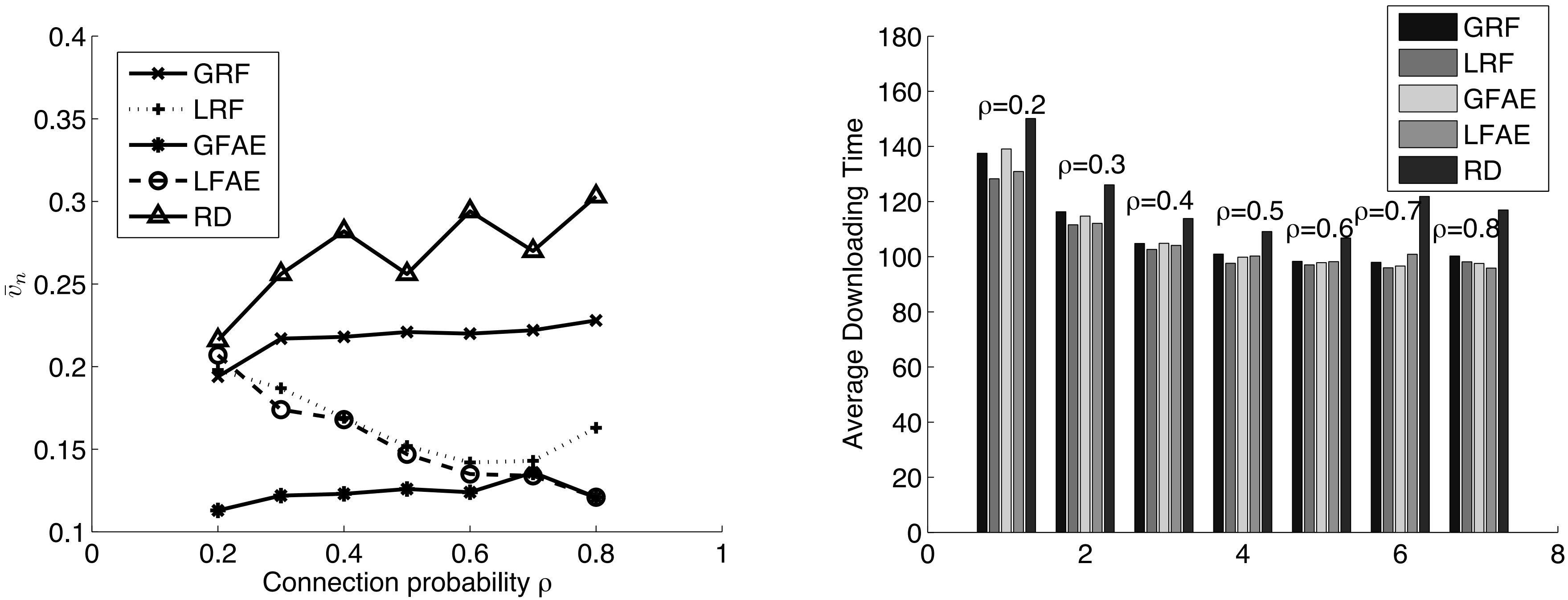
- **Choose chunk i probabilistically according to:**

$$\sigma_i = \frac{\Delta h_i}{\sum_{\forall \Delta h_j > 0} \Delta h_j}.$$

$$\Delta h_i = \begin{cases} \frac{\partial V}{\partial h_i} = \frac{2(\bar{h} - h_i)}{M} & \text{if } h_i \leq \bar{h} \\ 0 & \text{otherwise.} \end{cases}$$

Experiments

- Low bandwidth:



(a) Availability, $B = 4.5$

(b) Average downloading time, $B = 4.5$

Experiments

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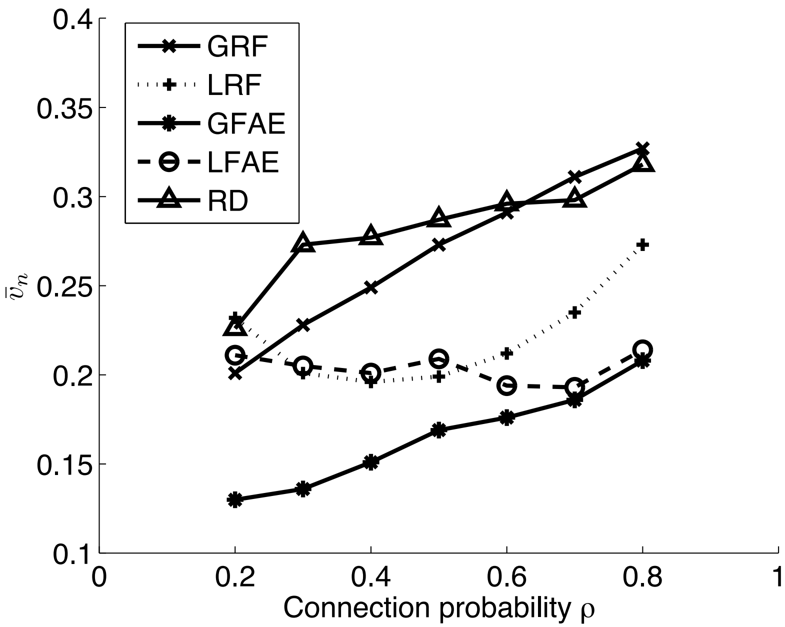
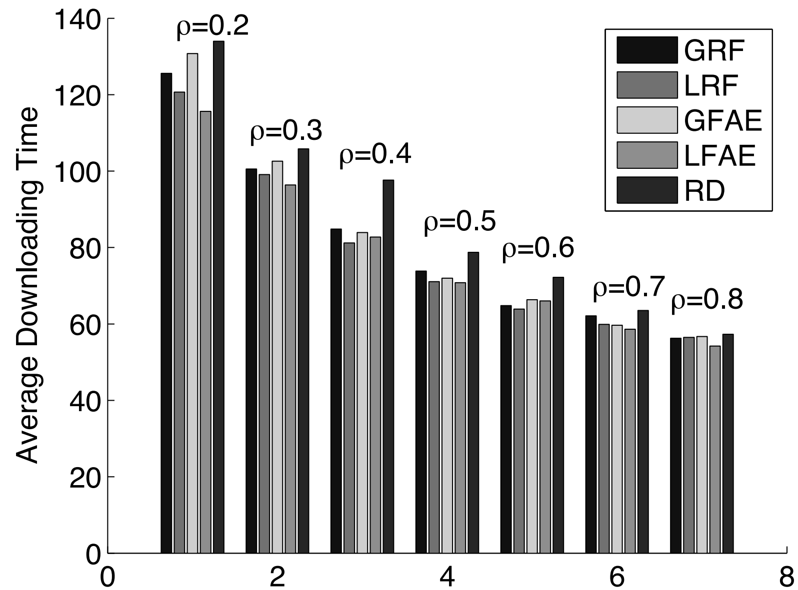


(a) Availability, $B = 12$



(b) Average downloading time, $B = 12$

Conclusion

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- **Propose the analytical model to understand BT**

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- **Propose the analytical model to understand BT**
  - **On Throughput,**
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  - **Sensitivity analysis on different system parameters.**
- **Extend the model to consider peers**

Conclusion (Cont.)

- Validate the analytical result with extensive simulation (our model is more accurate than the Qiu's model)
- Propose new approach on chunk selection algorithm to enhance file availability