Threshold Models of Collective Behavior[1]

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Models of collective behavior are developed for situations where actors have two alternatives and the costs and/or benefits of each depend on how many other actors choose which alternative. The key concept is that of "threshold": the number or proportion of others who must make one decision before a given actor does so; this is the point where net benefits begin to exceed net costs for that particular actor. Beginning with a frequency distribution of thresholds, the models allow calculation of the ultimate or "equilibrium" number making each decision. The stability of equilibrium results against various possible changes in threshold distributions is considered. Stress is placed on the importance of exact distributions for outcomes. Groups with similar average preferences may generate very different results; hence it is hazardous to infer individual dispositions from aggregate outcomes or to assume that behavior was directed by ultimately agreed-upon norms. Suggested applications are to riot behavior, innovation and rumor diffusion, strikes, voting, and migration. Issues of measurement, falsification, and verification are discussed.

BACKGROUND AND DESCRIPTION OF THE MODELS

Because sociological theory tends to explain behavior by institutionalized norms and values, the study of behavior inexplicable in this way occupies a peripheral position in systematic theory. Work in the subfields which embody this concern—deviance for individuals and collective behavior for groups—often consists of attempts to show what prevented the established patterns from exerting their usual sway. In the field of collective behavior, one such effort involves the assertion that new norms or beliefs "emerge"

[1] This report is heavily indebted to three co-workers. Christopher Winship is responsible for important aspects of the formalization, Douglas Danforth and Bob Phillips have carried out a large part of the mathematical analysis and computer programming. All have given valuable substantive help as well. Specific contributions are footnoted in the text. I have also benefited from the thoughtful comments of participants in seminars and colloquia I have given at Harvard, Wesleyan, Columbia, Stanford, Stony Brook, UCLA, and Temple. The work has been done mainly in the stimulating atmosphere of the Center for Advanced Study in the Behavioral Sciences, where the criticisms of Robert Axelrod, Brian Barry, and Arthur Goldberger led to improvements. The center's support was made possible in part by the Andrew Mellon Foundation and the National Science Foundation. Partial support for the research was provided by a sabbatical leave from Harvard University and by National Science Foundation grant SOC 76-11185 to the author.

in situations where old ones fail or few precedents exist (Turner and Killian 1957; Smelser 1963).

Such arguments are an advance over crude psychologizing about crowds' stripping away the "veneer" of civilization from their participants. But I will argue here that knowing the norms, preferences, motives, and beliefs of participants in collective behavior can, in most cases, only provide a necessary but not a sufficient condition for the explanation of outcomes; in addition, one needs a model of how these individual preferences interact and aggregate.

Because theories oriented to norms lack such a model, they end up assuming, implicitly, a simple relation between collective results and individual motives: that if most members of a group make the same behavioral decision—to join a riot, for example—we can infer from this that most ended up sharing the same norm or belief about the situation, whether or not they did so at the beginning.

The models I will describe, by contrast, take as the most important causal influence on outcomes the *variation* of norms and preferences within the interacting group. It will be clear even in the simplest versions of these models that the collective outcomes can seem paradoxical—that is, intuitively inconsistent with the intentions of the individuals who generate them. This possibility is foreclosed if we insist that collective outcomes reflect norms, whether old or new, of most of the participants. Further, once we abandon the definition of collective behavior situations as those in which people develop new norms or abandon existing ones, the range of situations which can be considered broadens. Thus, these models can be applied to processes not usually called "collective behavior," such as voting, residential segregation, diffusion of innovations, educational attainment, strikes, migration, and markets—as well as the more typical processes of crowd behavior and social movements.

It is best to stress at the outset what is not attempted. These models treat the aggregation of individual preferences; they do not consider how individuals happen to *have* the preferences they do. That very important question is outside the main concern of this paper. I begin with preferences and go from there. Most existing literature, by contrast, channels its main effort into determining how norms, motives, and preferences are caused and assumes that nothing more need be done to explain collective behavior. I maintain instead that once these are known, there is still a great deal to be done, and that outcomes cannot be determined by any simple counting of preferences. This will be particularly clear in cases where a very small change in the distribution of preferences generates a large difference in the outcome. Analysis focusing only on determination of preferences could not explain such a phenomenon.

Threshold Models of Collective Behavior

The models of this paper treat binary decisions—those where an actor has two distinct and mutually exclusive behavioral alternatives. In most cases the decision can be thought of as having a positive and negative side—deciding to do a thing or not to, as in deciding whether to join a riot—though this is not required for the formal analysis. A further requirement is that the decision be one where the costs and benefits to the actor of making one or the other choice depend in part on how many others make which choice. We may take riots as an example. The cost to an individual of joining a riot declines as riot size increases, since the probability of being apprehended is smaller the larger the number involved (see, e.g., Berk 1974).

The individuals in these models are assumed rational—that is, given their goals and preferences, and their perception of their situations, they act so as to maximize their utility. Individual differences are a main focus of the models. Different individuals require different levels of safety before entering a riot and also vary in the benefits they derive from rioting. The crucial concept for describing such variation among individuals is that of "threshold." A person's threshold for joining a riot is defined here as the proportion of the group he would have to see join before he would do so. A "radical" will have a low threshold: the benefits of rioting are high to him, the cost of arrest, low. Some would be sufficiently radical to have a threshold of 0%—people who will riot even when no one else does. These are the "instigators." Conservatives will have high thresholds: the benefits of rioting are small or negative to them and the consequences of arrest high since they are likely to be "respectable citizens" rather than "known rabble-rousers." Thresholds of 80% or 90% may be common, and we may allow for those individuals who would not join under any circumstances by assigning them a threshold of 100%.

It is not necessary, in fact, to be able to classify a person as radical or conservative from his threshold, and one strength of the concept is that it permits us to avoid such crude dichotomies. Since a threshold is the result of some (possibly complex) combination of costs and benefits, two individuals whose thresholds are the same may not be politically identical, as reflected in the popular expression, "strange bedfellows." The threshold is simply that point where the perceived benefits to an individual of doing the thing in question (here, joining the riot) exceed the perceived costs.[2]

[2] I have adapted the idea of behavioral thresholds from Schelling's models of residential segregation (1971*a*, 1971*b*, 1972), where thresholds are for leaving one's neighborhood, as a function of how many of one's own color also do so. The present paper has Schelling's aim of predicting equilibrium outcomes from distributions of thresholds but generalizes some features of the analysis and carries it in somewhat different directions. The model has some resemblance also to one known in psychology as "behavioral contagion" (for a review see Wheeler 1966). In Wheeler's formulation, the cost-benefit analysis described here is seen as

The focus of this paper is the formal model and *not* the substantive question of riot behavior. In describing the model I will nevertheless usually talk about "riot thresholds" because this is a convenient and colorful illustration; but it has no special conceptual status, and the reader should keep in mind that the analysis is meant to apply to any appropriate binary decision. Before beginning formal analysis, therefore, I will suggest a catalog of other binary-choice situations where threshold models could be applied.

1. Diffusion of innovations. Women in Korean villages may be wary of adopting birth control devices and wait to do so until some proportion of their fellow villagers do. Different women will have different thresholds, depending upon their education, age, husband's opinions, position in a hierarchy of informal leadership, or personal tastes (see Rogers 1975; Dozier 1977).

2. Rumors and diseases. In order to spread a rumor, one must hear it from another person. But people vary in their credulity and some may need to hear it from more than one other before they will believe it enough to spread it. These levels of credulity are the same as thresholds. A formally identical situation is the spread of a disease, where credulity is replaced by "vulnerability": people differ in how many infecteds they must be exposed to before they too catch the disease.

3. Strikes. Workers deciding whether to strike will attend carefully to how many others have already committed themselves, since the cost of being one of a small number of strikers is high, especially in a vulnerable employment situation. One would thus expect teachers without tenure to have higher strike thresholds than those with tenure.

4. Voting. One's decision to vote for a particular candidate may depend heavily on how many others have already decided to do so, partly because

an "approach-avoidance conflict," and "contagion" occurs at the point where observing another individual's behavior pushes the approach tendency above the avoidance tendency —nearly equivalent to the definition above of threshold as the point at which benefits exceed costs. No consideration is given, however, to how many individuals one might need to observe before this point is reached, or of cumulative effects of those observed before the final person. In the broader context of threshold models, the idea of "contagion" seems inappropriate, since much more is involved than mere imitation of the last person observed. There is also some similarity between the present models and models used in epidemiology (as in Bailey 1976), the diffusion of information (Bartholomew 1967) and innovations (Hamblin, Jacobsen, and Miller 1973), and the evolution of behavior in groups over time (Coleman 1965, chaps. 10 and 11). To develop these analogies in more detail would require that (1) my models, expressed below as difference equations in discrete time, be translated into differential equations in continuous time, and that (2) some way be found to introduce the "threshold" concept into these other models, which generally do not stress individual differences (cf. especially Coleman's discussion of "heterogeneity"). While some work in this direction has been accomplished, it is incomplete and could not be adequately presented in a brief way. Hence, it is deferred to future publications.

of social influence, partly because one does not want to waste one's vote. One outcome of this situation is what we call "bandwagon effects."

5. Educational attainment. The decision to go to college depends in part on what proportion of one's cohort does so. This is partly because of peer-group influence, partly because a large attendance in one's cohort raises the general level of credentials in the labor market, making it more difficult to find suitable employment without a college degree (see, e.g., Berg 1970).

6. Leaving social occasions. We have all had the experience of sitting impatiently at a boring lecture, unable to leave because not enough others have yet done so. People vary in their thresholds for leaving lectures, cocktail or dinner parties, or other occasions. The variation is composed partly of personality traits—politeness, timidity—and partly of the press of other obligations.

7. Migration. It is well known that migration decisions depend heavily on those of others, as in "chain migration" (MacDonald and MacDonald 1964). Those with low migration thresholds are likely to have more psychological and economic resources than those who migrate later.

8. Experimental social psychology. Experiments on conformity achieve varying results depending on the number of confederates introduced (Asch 1956; Milgram, Bickman, and Berkowitz 1969). "Risky shift" experiments could be reanalyzed to study the time sequence in which people change from less to more risky alternatives (see Pruitt and Teger 1971). Situations of bystander intervention could be described by means of "helping thresholds."

Equilibrium Outcomes in Simple Threshold Models

For all the examples above, the aim of the formal model presented here is the same: to predict, from the initial distribution of thresholds, the ultimate number or proportion making each of the two decisions. Mathematically, the question is one of finding an equilibrium in a process occurring over time. A simple example will make the procedure clear.

Imagine 100 people milling around in a square—a potential riot situation. Suppose their riot thresholds are distributed as follows: there is one individual with threshold 0, one with threshold 1, one with threshold 2, and so on up to the last individual with threshold 99.[3] This is a uniform distribution of thresholds. The outcome is clear and could be described as a "bandwagon" or "domino" effect: the person with threshold 0, the "instigator," engages in riot behavior—breaks a window, say. This activates the person with threshold 1; the activity of these two people then activates the person

[3] Depending on the substantive situation, thresholds may usefully be described as either proportions or absolute numbers. The mathematical analysis is the same in either case. For convenience, numerical examples given in this paper use groups of 100 people, so the reader may think of thresholds as proportions or numbers.

with threshold 2, and so on, until all 100 people have joined. The equilibrium is 100.

Now perturb this distribution as follows. Remove the individual with threshold 1 and replace him by one with threshold 2. By all of our usual ways of describing groups of people, the two crowds are essentially identical. But the outcome in the second case is quite different—the instigator riots, but there is now no one with threshold 1, and so the riot ends at that point, with one rioter.

Even this simple-minded example makes the main point suggested earlier: it is hazardous to infer individual dispositions from aggregate outcomes. Newspaper reports of the two events would surely be written as, in the first case, "A crowd of radicals engaged in riotous behavior"; in the second, "A demented troublemaker broke a window while a group of solid citizens looked on." We know, however (since we constructed the example), that the two crowds are almost identical in composition; the difference in outcome results only from the process of aggregation, and in particular from the gap in the frequency distribution in the second case.

The reader may want to try out the example as well on the cases of innovation adoption, rumor or disease spreading, strikes, voting, going to college, leaving social occasions, migration, or conformity. Bandwagon effects can be imagined for each, but the sensitivity of such effects to exact distributions of preferences is rarely appreciated. Threshold models may be of particular value in understanding situations where the average level of preferences clearly runs strongly in favor of some action, but the action is not taken. The usual sociological models of action have limited value in such cases.

It is possible to give a mathematically exact account of how one goes from a frequency distribution of thresholds to an equilibrium outcome.[4] Denote thresholds by x, the frequency distribution by $f(x)$, and the cumulative distribution function (c.d.f.) by $F(x)$—where the c.d.f. indicates the proportion of the population having threshold less than or equal to x. Call the proportion of the population who have joined a riot by time t (using discrete time periods) $r(t)$. Suppose we know $r(t)$ for some t—for example, suppose we know that after two time periods ($t = 2$) 60% of the crowd has joined in. Then what proportion of the crowd will be rioting at $t = 3$? It must be, by definition of thresholds, exactly that proportion of the crowd whose thresholds are less than or equal to 60%. It follows immediately that the process is described by the difference equation: $r(t + 1) = F[r(t)]$.

Where the frequency distribution has a simple form, the difference equation can be solved explicitly to give an expression for $r(t)$ at any value of t. Then, by setting $r(t + 1) = r(t)$, the equilibrium outcome may be

[4] This analysis and that of fig. 1 are due to the efforts of Christopher Winship.

found. Where the functional form is not simple, the equilibrium may nevertheless be computed by forward recursion. In this simple version of the model, where no provision has been made for "removal" of participants, oscillatory behavior of $r(t)$ is not possible, and an equilibrium will always be reached.

Some graphical observations show that equilibrium points can be computed without manipulating difference equations or engaging in forward recursion. In figure 1, we graph thresholds (x) against the c.d.f. [$F(x)$]. Suppose, as before, that $r(t)$ is known. Since $r(t + 1) = F[r(t)]$, we may find the proportion rioting in the next time period by following the leftmost arrow from $r(t)$ to the point immediately above it on the c.d.f. To locate this point again on the x-axis, we follow the horizontal arrow to the 45° line, $F(x) = x$. This procedure can then be repeated to find $r(t + 2) = F[r(t + 1)]$, and so on. For the c.d.f. drawn in figure 1, we can see that the

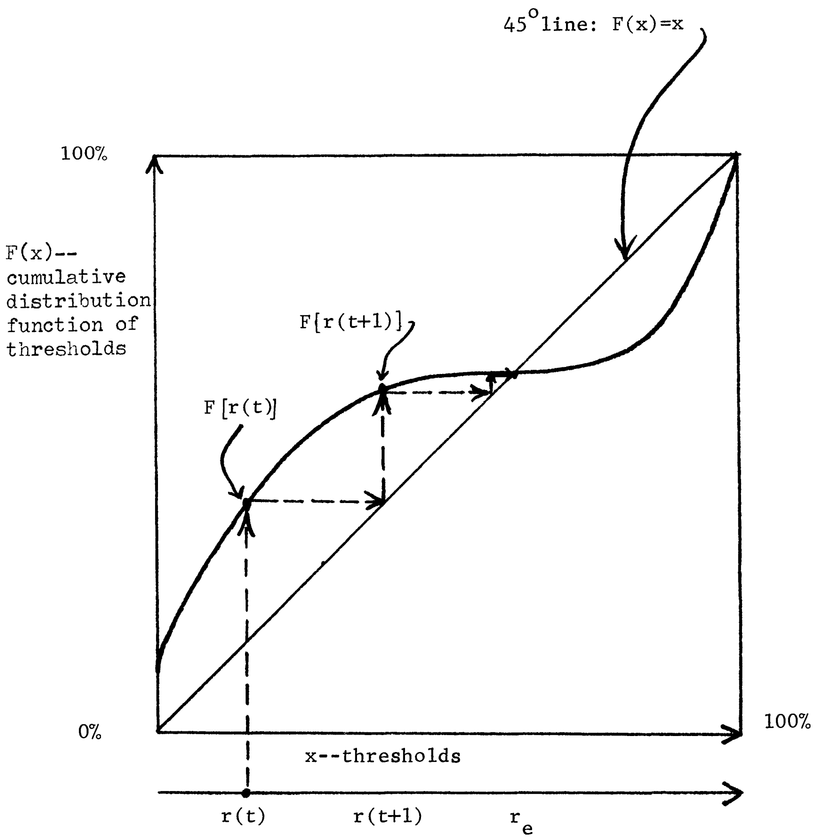

FIG. 1.—Graphical method of finding the equilibrium point of a threshold distribution. $r(t)$ = proportion having rioted by time t.

horizontal length of the arrow goes to zero, and $r(t)$ goes to a limiting value called r_e—the equilibrium point. That limit is the point where the c.d.f. first crosses the 45° line from above. Algebraically, the point is denoted by the equation $F(r) = r$.

Without empirical or theoretical reason to expect one particular distribution or another, it is not fruitful to pursue extensively the behavior and equilibria of large numbers of functional forms. However, I will present some results obtained from normal distributions of thresholds, using the equilibrium analysis of figure 1. The normal frequency distribution is of interest here because we may take it to be characteristic of populations where no strong tendencies of any kind exist to distort a distribution of preferences away from its regular variation about some central tendency. Yet, the results obtained are striking and counterintuitive, showing that paradoxical outcomes are not limited to special distributions such as the uniform.[5]

Consider, again, 100 people; let their thresholds now be normally distributed, with mean threshold equal to 25. (Those thresholds below zero may be regarded, for practical purposes, as equivalent to zero. Theoretically they may be seen as degrees of radicalism which may have ideological significance but lead to the same action. Similar comments apply to thresholds above 100.) We may now ask what the effect is on the equilibrium outcome of varying the standard deviation of our normal distribution, leaving the mean fixed. The surprising result is graphed in figure 2, a plot of r_e against σ, the standard deviation. Up until a critical point, σ_c, the equilibrium number of rioters increases gradually to about six. Then after this point, approximately 12.2, the value of r_e jumps to nearly 100, after which it declines. (The limiting value, as σ increases without bound, is 50, since eventually all the area to the right of the mean can be seen as beyond 100, all the area to its left below 0.)

Mathematically, this is easily explained. Equilibrium is found by noting the first intersection of the c.d.f. with the 45° line, from above. The normal c.d.f. may intersect the line either three times, twice, or once. For values of σ below σ_c, the first intersection from above is at a low point and is followed by one from below and, later, by another from above. At the critical point, σ_c, the first two intersections are combined in a point tangent to the 45° line, and there is one further intersection above. After this point, the only intersection occurs near 100, dropping off gradually as the probability density flattens.

This mathematical account, however, has no substantive companion. There is no obvious sociological way to explain why a slight perturbation of the normal distribution around the critical standard deviation should have a wholly discontinuous, striking qualitative effect. This perturbation

[5] The analysis of normal distributions of thresholds is due to the efforts of Bob Phillips.

might correspond to a minor fluctuation in the composition of a crowd, or to some change in the situation which altered the distribution of thresholds a bit—a cause which would seem so insignificant in relation to its effect that causal attribution would never be made. This is particularly the case since sociological theory is not at all oriented to analyzing the effects of changes in exact distributions of properties but concentrates, rather, on the effects imputed to average values. This example shows again how two crowds whose average preferences are nearly identical could generate entirely different results. As in the cases described earlier, we would be highly unlikely to infer accurately the structure of preferences that led to the outcomes without an explicit model of the aggregation process.

The Stability of Equilibrium Outcomes

Effects of friendship and influence.—The above discussion of equilibrium outcomes points up the need for systematic treatment of the stability of the equilibrium which follows from some given distribution of thresholds. For dynamic analysis this is indispensable, since a variety of influences may intervene in real situations to modify existing distributions. Whether these influences have small or large effects depends on these stability considera-

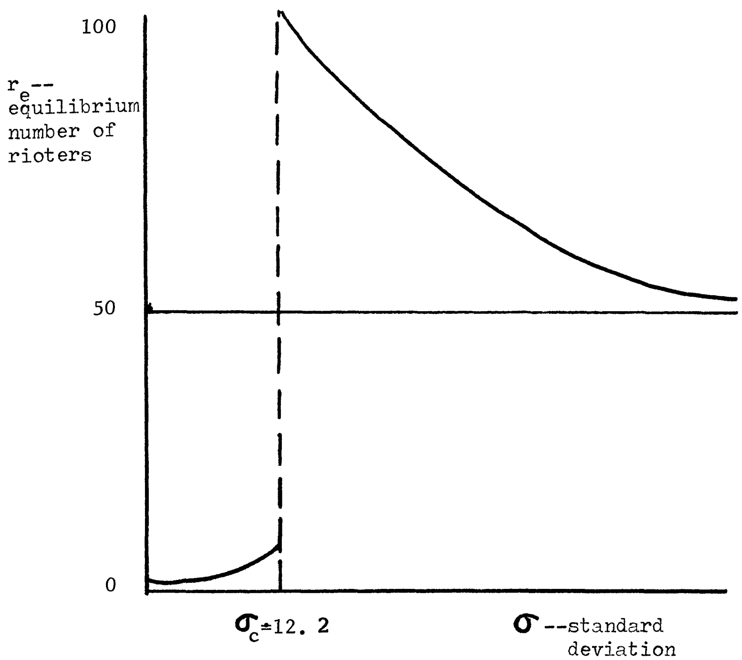

FIG. 2.—Equilibrium number of rioters plotted against standard deviation of normal distributions of thresholds with mean = 25, N = 100.

tions. One of the most obvious ways for the distribution to change is for some individuals to enter or leave the situation. We can take the varying of standard deviations in normal distributions or the changing of a uniform to a perturbed uniform distribution, described above, as a way of thinking about what happens to a distribution when such entry and departure occur. We find, then, that the true uniform distribution and the normal one near its critical standard deviation have highly unstable equilibria.

An eventual aim of the analysis of threshold models is to develop mathematical procedures for assessing the stability characteristics of any distribution's equilibrium under a variety of possible perturbations. In this section and the next, I discuss two particular factors which may play important roles in changing the effects of threshold distributions: social structure and the spatial/temporal dispersion of social action.

By "social structure" I mean here only that the influence any given person has on one's behavior may depend upon the relationship. Take a simple case, where the influence of friends is twice that of strangers, and assume that thresholds are given in terms of reaction to strangers. Consider an individual with threshold 50% in a crowd of 100, where 48 individuals have rioted and 52 have not. In the absence of social structure, such an individual would not be activated. But if he knows 20 people in this crowd of whom 15 have already joined the riot, then each friend is to be counted twice. Instead of "seeing" 48 rioters and 52 nonrioters, our subject "sees" $[(15 \times 2) + (33 \times 1)]$ rioters and $[(5 \times 2) + (47 \times 1)]$ nonrioters, leading him to form a ratio not of 48/100 but of 63/120 = .525. What we may then call the "perceived proportion of rioters" in the previous time period now exceeds his threshold, and he will join.

For any given situation, this procedure allows the computation of equilibrium results by forward recursion as long as we have the distribution of thresholds, a sociomatrix, and a weight for each sociomatrix cell, corresponding to how much more influence i has on j than a stranger would have. To make more general statements about the effects of social structure on outcomes, we need to say more systematically what parameters of social structure are of interest. Does a high or a low density of friendship ties, for instance, have the greater effect in modifying the equilibrium result of some particular distribution of thresholds? This question could be posed as well for lesser or greater weights.

For a given friendship density, however, even a moderate number of people generates an almost limitless number of possible sociomatrices. If we fix a threshold distribution and make the weights of friends all identical, each such matrix yields an equilibrium outcome, and the set of matrices gives a frequency or probability distribution of equilibria. Repeated attempts to derive these distributions analytically have failed, yielding only unmanageable convolutions. Some partial results are available, however, if we do not require friendship to be symmetric. We may pose the null hypothesis that introduction of social structure makes no difference in the equilibrium generated by a given threshold distribution.

Consider the perturbed uniform distribution described above: one person with threshold 0, two with threshold 2, one with 3, one with 4 . . . one with 99. The null hypothesis is an equilibrium of one—only the instigator with threshold 0 would riot. Our analysis indicates that the null hypothesis becomes increasingly improbable as the weight attached to friends' behavior increases;[6] this seems intuitively reasonable. The results for acquaintance volume are more surprising; the largest effects occur where people know, on the average, about one-quarter of the rest of the group—a moderate level of friendship. The explanation is neither intuitively nor analytically obvious and will require further study.

To get fuller results on overall probability distributions of equilibria, we have had to construct computer simulations of the process in which the computer fixes parameters, constructs one sociomatrix after another from the class to be sampled, and generates an equilibrium by forward recursion. Simulations of the perturbed uniform distribution yield results quite close to our analytical ones and indicate that the symmetry of ties has little effect on outcomes. Simulations allow us to look not only at the null hypothesis of no change but also at the typical extent of change. For this particular distribution, even when the null hypothesis of no change is false, the equilibrium changes very little and rarely exceeds five to 10 rioters. Thus, the underlying outcome, one rioter, is relatively stable in the face of social structural influences. By contrast, simulation of social structural effects on the true uniform distribution of thresholds shows that its equilibrium of 100 rioters is unstable against almost any kind of social structural influence. For most combinations of weights and acquaintance volume tested, the modal equilibrium result is one rioter.

The general goal of such analysis is to specify the impact of social structure on collective outcomes. Most collective-behavior literature proceeds as if the groups discussed contained only people who are strangers to one another (an exception is Aveni 1977). I believe that social structure within interacting groups has important but complex relationships to results. When threshold distributions have very stable equilibria it may make very little difference; when these equilibria are unstable, however, the effects of social structure may overwhelm those of individual preferences. Sorting out the circumstances under which one effect is more important than another will improve our understanding of both social structure and collective behavior.

[6] These results and the subsequent computer simulations are the work of Douglas Danforth. More detail on both is presented in an earlier draft of this paper, available from the author.

Spatial and temporal effects.—Social structure is one reason why the simple form of threshold models may not provide an adequate account of events. Another is that the simple model makes an implicit assumption of complete connectedness which is often inappropriate: that each individual is responsive to the behavior of *all* the others, regardless of the size or spatial or temporal dispersion of the aggregation. Margaret Stark et al. (1974), for example, in an analysis of the 1965 Watts riot, report that rather than being a single incident it consisted of more than 1,850 separate cases of riot action in five days of rioting over a wide area.

Modeling the effects of spatial and temporal dispersion on equilibrium outcomes presents greater mathematical difficulties than those described in the previous sections, and progress has been slower. A few simple results will suggest, however, that interesting possibilities arise.

For simplicity, imagine a large population in some city with a distribution of riot thresholds equal to the uniform distribution discussed above: 1% of the population has threshold 0%, 1% has threshold 1%, 1% has threshold 2% . . . 1% has threshold 99%. Suppose also that when a crowd gathers it consists of a random sample from this large population and that its size is always 100. Recall that when a group with this *exact* distribution of thresholds gathers, the (deterministic) equilibrium is that everyone riots. But sampling variability changes this equilibrium conclusion.[7] If on one of these occasions, for example, the crowd that gathers contains no zero percenter (instigator), the resulting equilibrium is zero. The probability of this occurrence can be computed if we think of the 100 crowd members as having been drawn by Bernouilli trials, with probability of success equal to .01 ("success" = drawing a zero percenter). The probability of no successes in 100 trials is then $(1 - P)^{100} = .37$. Further, the chance of drawing one zero percenter but no one percenters—so that the equilibrium result is one rioter—is the product of the two probabilities, and thus equals $[\binom{100}{1}(.01)(.99)^{99}] \times (.99)^{100} = .14$. This means that in over half the cases ($.37 + .14 = .51$) the equilibrium result is either no rioters or one rioter. We see in still another way how vulnerable the equilibrium given by a uniform distribution of thresholds is to perturbation. The example suggests that in cities where a fairly constant distribution of riot thresholds exists the outcome of one crowd may nevertheless differ radically from that generated by another, at some earlier or later time, for reasons that have nothing to do with differences between the occasions or intervening events but involve only sampling variability. This will occur if the underlying distribution yields an equilibrium which is easily disrupted by changes. It then also

[7] While it may stretch the imagination a bit to argue that all crowds are simple random samples from the population at risk, there is a good deal of evidence for the more-or-less random character of casually forming groups of people (see Coleman 1965, pp. 361–75; White 1962; Cohen 1971).

follows that in two cities which have the same underlying distribution of riot thresholds one may experience a large riot and the other not, for reasons which do not reflect intercity differences.

Spilerman attempted to determine what characteristics of cities might have contributed to the number and severity of racial disorders experienced from 1961 to 1968 (Spilerman 1970, 1971, 1976). For both dependent variables, the only city characteristics which had an important impact, in stepwise multiple regression, were the absolute size of the black population and, much less significant, a dummy variable for region: South or non-South. Since Spilerman's main concern was to argue that particular city conditions did not affect the probability of riots—that the phenomenon was a national one—he spent little time discussing the substantial correlation with absolute black population, suggesting only that this variable "relates directly to the ability of the Negro community to mobilize a disorder and also to the number of incidents occurring in a ghetto which might precipitate a disturbance. . . . Disorder proneness is inherently a personal attribute, a response to factors which are exogenous to the community but visible in all ghettos. The community propensity, in this formulation, is an aggregate of the individual values and would therefore reflect the numerical size of the Negro population" (1970, pp. 643–44).

The temporal sampling variability discussed above suggests a mechanism which could explain Spilerman's correlation but would involve more than only "personal attributes." Suppose that before an incident is reported as a "riot" the equilibrium number of rioters must reach some level, and that a city has, each time a crowd gathers, the same probability of reaching this particular equilibrium. (Such a probability is determined by the underlying distribution of riot thresholds from which each crowd is drawn.) If this probability is, say, .10, and we argue as Spilerman does that the larger the black population, the larger the number of incidents which occur, then we may think of each incident as a Bernouilli trial with probability of success (a large riot) of .10. It is then clear that the expected number of large riots is a function of the number of incidents, since the mean of the binomial is the number of trials multiplied by the probability of success. In a community sufficiently small that only one "trial" occurred over a specified time period the chances are 90% that no large riot would occur. But in a larger city where 10 incidents occurred the chance of no riot falls to $(.90)^{10} = .35$, even though the distribution of thresholds is the same.

Sampling variability would have spatial as well as temporal effects. One can think of multiple samples from an underlying population as representing discrete clusters of individuals over some large area during the same time period. Then it follows that, despite being drawn from the same distribution, to the extent that the equilibrium is an unstable one there would be wide divergence from one place to another in the local outcome. A next natural question would concern movement from one cluster to another and its effect on the overall number of rioters. One might ask, for example, what level of movement among clusters would have the most incendiary effect. The answer is not straightforward, since too much movement out of a cluster which had reached a high equilibrium may have the effect of deactivating some rioters, which would then deactivate others, until a new and possibly much lower equilibrium was reached—a reverse bandwagon. Thus, for some threshold distributions small movements among clusters may have greater effects than large ones. Much more mathematical work is needed to make this statement precise.

The introduction of spatial considerations makes the model more reasonable for large riots and generally for fragmented situations where there is some modest level of connection among fragments. A natural situation to model in this way would be the adoption of innovations where the main decisions take place in fairly discrete units and where there is some movement among units, for example, the adoption of family planning by women in Korean villages where there is some intervillage migration (Rogers 1975). The general import of these models is that threshold distributions, friendship structures, and migration patterns could have more impact on level of adoption ("equilibrium" results) than any features of the programs themselves. Analysts of family planning are often frustrated when the same program has different outcomes in areas where the average preferences are nearly identical. Threshold models may explain how this can occur. Another situation which could be modeled with spatial considerations is recruitment into political parties and spread of a party across a country. Relevant thresholds are those for joining the party, for example, the spread of Nazi party membership in Weimar Germany.

SOME THEORETICAL AND EMPIRICAL CONSIDERATIONS

Having sketched threshold models of collective behavior, I want to explore further their conceptual underpinnings, their relation to more usual explanations of collective behavior, and the possibilities for falsifying or verifying such models.

Thresholds, Game Theory, and Norms

Threshold models share with game-theoretic models the assumption of rational actors with complete information. Much recent literature on collective behavior has reacted against the older notion that irrationality is the key to explanation; theorists as diverse as Tilly (1975) and Banfield (1970) agree that collective behavior often results from rational, sometimes calculated, action.

Since the best-developed formal account of rational action in situations of mutual interdependence is given by game theory (see Luce and Raiffa 1957), and since promising work has been done on game-theoretic models of collective behavior, it is worth saying what I think to be the advantages of threshold models.

Roger Brown's pioneering account (1965) reduced many typical collective behavior episodes—such as the stampede from a burning theater—to a Prisoners' Dilemma game (see Luce and Raiffa 1957, pp. 95–97); he collapsed the actions of a large number of people into the analysis of a two-person game in which each person "plays" against all the others taken collectively. Economists employ a similar strategy in explaining bank failures, refusals of farmers to cut production voluntarily, and inability of citizens to organize to achieve "public goods" (see Samuelson 1967, p. 12; Luce and Raiffa 1957, p. 97; Olson 1965).

All these examples have considerable intuitive appeal since they display situations where rational individual action, in pursuit of well-defined preferences, leads to outcomes undesirable to the actors and surprising, given their intentions. But reduction to a two-person game is possible only if all the actors are homogeneous in their preferences. A general analysis of collective behavior cannot be satisfied with such a severe limitation, since many situations of interest involve participants with widely different goals. While a body of theory does exist on games involving more than two people ("*n*-person games"), the analytical situation is far less satisfactory than for the two-person situation. The larger the number of distinctly different actors, the more difficult it becomes to characterize the outcome without introducing arbitrary assumptions (see Rapoport 1970).

Another difficulty is that game theory typically assumes that all actors' decisions are made simultaneously; no one's decision is *contingent* on anyone else's previous one. Both difficulties can be seen in Berk's stimulating game-theoretic analysis of a riot situation (Berk 1974). In his treatment, even the presence of two distinct sets of actors (militants and moderates) leads to considerable complication; yet it is hard to imagine that division into these two categories accurately depicts the range of variation present in the crowd. Further, Berk's adherence to the principle of simultaneous decisions leads him to try to explain the outcome solely in terms of events that occur *before* any riot action begins—namely, crowd members' attempts to change one another's perceived payoffs for various outcomes (p. 364). The decision of an individual to act—that is, join the riot—"rests on expectations of what others will do, [therefore] enough crowd members must arrive at parallel assessments which make action for all a good bet before activity is likely to begin" (p. 368). Here Berk not only forecloses the possibility of evolution of the process *after* riot activity starts but also requires a convergence among crowd members before it begins which is reminiscent of the notion of "emergent norms." In fact, Berk argues explicitly that "the gaming perspective fills important gaps in emergent norm theory"—namely, explaining the origin of such norms (p. 372).

Threshold models take the two elements of collective behavior which game theory handles only with difficulty and makes them central: substantial heterogeneity of preferences and interdependence of decisions over time. This is possible because the n-dimensional payoff matrix of game theory is replaced by a one-dimensional vector of thresholds, one for each actor. This allows enormous simplification in the ensuing analysis.

Like all simplifications, this one carries a cost. The payoff matrices of game theory allow us to investigate, for any particular actor, which outcome maximizes his utility and whether outcomes are Pareto optimal for the whole set of actors. Threshold analysis does not permit this. When an individual is activated because his threshold is exceeded, he acts so as to maximize his utility under existing conditions. The resulting equilibrium may or may not maximize anyone's overall utility. From the distribution of thresholds alone, nothing can be said about this. Thresholds do not give information about the utility to an individual of each possible equilibrium outcome.[8]

The equilibrium reached may well be suboptimal for most actors. Consider, for example, Matza's account of the behavior of delinquent boys whom he interviewed (1964, chap. 2). Most did not think it "right" to commit illegal acts or even particularly want to do so. But group interaction was such that none could admit this without loss of status; in our terms, their threshold for stealing cars is low because daring, masculine acts bring status, and reluctance to join, once others have, carries the high cost of being labeled a sissy or "faggot." Bandwagon effects occur like those of the formal models. But note that the suboptimality of the outcome for most boys would not be known to us if all we had was information on their thresholds; to know this, we require Matza's interviews as well.

This example shows also that thresholds are quite different from "norms"—another construct frequently invoked in the explanation of collective behavior. The boys act because their threshold is exceeded, and their utility is maximized, given the situation, by joining in the criminal activity. But in so doing they act contrary to norms they actually hold. That this is so indicates not that norms are irrelevant, but rather that they are only one causal influence on behavior and are not always decisive. The concept of threshold, then, is purely behavioral, connoting nothing about what the actor thinks is the "right" thing to do.

[8] Schelling (1973) describes models of binary choice situations where full utility functions are given explicitly. Most of his analysis requires every actor to have the same function, but see the brief discussion of how one might relax this limitation (pp. 415–22). The time sequence of choices is not treated.

The Nature and Determinants of Thresholds

Thresholds are different from norms but result in part from them; Matza's boys would have even lower thresholds for delinquent activities, one supposes, if they did not feel they were wrong. Thresholds are also affected by most of the causal variables typically studied as determinants of individual behavior—background characteristics, social class, education, occupation, and social position; these all help establish the valuation given by an individual to different outcomes in a situation. The nature of the situation itself contributes to defining what outcomes are possible. Since behavior is partially determined by all the usual characteristics studied, it is not surprising that they have some moderate correlation with behavior. But if we take the threshold model seriously, it also follows that correlational studies will miss the dynamics of aggregation and thus be unable to provide more than this moderate level of correlation. In situations where we have, for example, two nearly identical distributions of thresholds which generate very different outcomes, correlational studies making predictions in the usual way from multiple-regression procedures will predict the outcome to be the same.

Thresholds are situation-specific. An individual's riot threshold is not a number that he carries with him from one riot to another but rather results from the configuration of costs and benefits, to him, of different behaviors in one particular riot situation. Inevitably, some situations will engage an actor more ideologically than others; one will seem more dangerous, one more exciting. What is argued is only that thresholds in many situations remain the same long enough for a predictable equilibrium to be reached.[9]

Great changes in behavior must not be confused with corresponding changes in threshold. We may imagine that individuals in a lynch mob are transformed by the situation—that their dispositions, values, and preferences are changed and deranged. The threshold model suggests instead that there is continuity of behavioral *dispositions* before, during, and after the lynching process—that mob members bring certain contingent dispositions to act into the situation (their lynching thresholds), and that while behavior changes, these do not.

This is not to say that there are not situations where individuals' central preferences and values (thus, most likely, their thresholds) do change. This may occur where people are subjected to great emotional shocks, as in religious revival meetings where people undergo "conversion experiences." But even this case is unclear; accounts of such meetings strongly suggest bandwagon effects, and those who undergo conversion often appear, on later analysis, to have been predisposed by a life of trouble and despair.

Thresholds may also change in the course of a situation because some-

[9] I am indebted to Lewis Coser for insisting on the implications of this point.

thing happens which changes the costs and benefits of the two possible decisions. In a riot, for example, the arrival of a heavy contingent of "law and order" forces may greatly increase the cost of participation; for those with a particular ideological orientation, however, this event may also increase the subjective benefits.

That thresholds may change because of emotional or situational changes does not invalidate the underlying model. It does, however, point up the importance of analyzing the stability characteristics of equilibria for threshold distributions. The persistence of riot behavior (or of any particular binary decision), in the face of changes which one might expect to have considerable impact on thresholds, may be difficult to explain if we do not understand that the initial threshold distribution had an equilibrium which was unusually resistant to perturbation.

The Falsification of Threshold Models

One may reasonably wonder whether there are not situations where the behavior of individuals cannot usefully be summed up and predicted by the proportions of others who engage in one or another of two possible behaviors. An extreme case is where individuals who appear to react to one another are actually all responding to an external influence. "Thus, if at the beginning of a shower a number of people on the street put up their umbrellas at the same time, this would not ordinarily be a case of action mutually oriented to that of each other, but rather of all reacting in the same way to the like need of protection from the rain" (Max Weber [1921] 1968, p. 23). But even Max Weber may be amended: there are surely some whose umbrella behavior is determined in part by that of others around them.

Here it is important to say that assigning someone a threshold of $x\%$ for raising his umbrella is not at all to say that the behavior of others is the most important influence on his behavior; it is only to make the statement that he will leave it closed until $x\%$ have opened theirs, then open it. The behavior of others may be only a marginal influence; but if the behavioral statement can be made, a threshold model can be constructed. A person in bad health, who detests being wet and wears a suit marked "dry-clean only," has strong nonsocial influences operating on his behavior. These are not ignored in the threshold model but rather contribute to the threshold itself: such a person's threshold will be lower, other things being equal, than that of someone not subject to such influences. Where the threshold model is of little interest, and may be said to have been, in effect, falsified, is where all or most thresholds are at 0% or 100%—that is, where most people's behavior is not, in fact, contingent on that of others.

A different difficulty arises if people do not make accurate judgments of the number or proportion of others who have made one or the other decision.[10] We may distinguish three different cases: (1) People may systematically misperceive the proportion of others who have, say, joined a riot, because of elements of the situation (ecological barriers) or personal characteristics (ideological "wishful thinking"). If this misperception is the same for all, as where everyone over- or underestimates by the same amount, the model is unchanged. Adjustments are not difficult if the level of misperception varies systematically with one's threshold—as when radicals overestimate and conservatives underestimate riot participation. If misperception is more random, the situation is more complex and the stability of the underlying equilibrium becomes particularly important. (2) Inaccurate judgments may occur because those who have decided to do something do not make their action public. What Merton has called "pluralistic ignorance" may occur where the costs of being known as one of a small number who, for example, adopt family planning, are seen as high. Thus, many thresholds may have been passed, but the actors who have them do not know this and hence do not act. Here, the threshold model is not only not falsified but is a valuable guide for the policymaker who wants to understand the forces which prevent full adoption of the innovation. (3) Individuals may be unable to make sufficiently fine distinctions for their true thresholds to have operational meaning. Someone with a riot threshold of 17% may be unable, for example, to distinguish among values between 15% and 20%; studies of human information processing suggest limits to the number of distinctions which can be made along one sensory dimension (Miller 1956). In such a case, the only threshold which has behavioral meaning is the lowest one which will be perceived by the individual as representing his true threshold. In this case, the operational threshold becomes 15% and enters the model in the usual way.

The threshold models described in this paper do require that one's reaction to others' behavior have a relatively simple form. Figure 3 graphs for some arbitrary individual, whose threshold is 38%, the net benefit to him (total benefit minus total cost) of joining a riot, for various levels of participation of the entire group. The threshold, as required by its definition, is the point where the net benefit first becomes positive. The curve is intentionally not monotonic, since the threshold concept does not require it to be. What the present models stipulate is that this curve not cross the x-axis more than once. If some individuals' curves do so, more complex models are needed. Suppose you are in an unfamiliar town and enter an unknown restaurant

[10] This paragraph has benefited particularly from comments of Robert Merton, Everett Rogers, Eugene Weinstein, and Larry Kincaid.

on Saturday evening at seven o'clock. Whether or not you decide to take a meal there will depend in part on how many others have also decided to do so. If the place is nearly empty, it is probably a bad sign—without some minimal number of diners, one would probably try another place. But the curve will cross the x-axis again at a later point—where the restaurant is so crowded that the waiting time would be unbearable. Some cautious individuals might join a riot when 50% of the others had but leave when the total passed 90% for fear that so large a riot would bring official reprisals. In principle, one may imagine the net-benefit curves of figure 3 crossing the x-axis any number of times; in practice, it is hard to think of examples where more than two seems reasonable. In the case of two, results can still be computed by forward recursion, but equilibria cannot be guaranteed and a somewhat different analytical apparatus would be needed. Empirically, if we found that aggregate behavior oscillated from one level of participation to another even though the situation seemed stable in its basic cost-benefit configuration, we might guess that at least some participants had net-benefit curves which crossed the x-axis more than once.

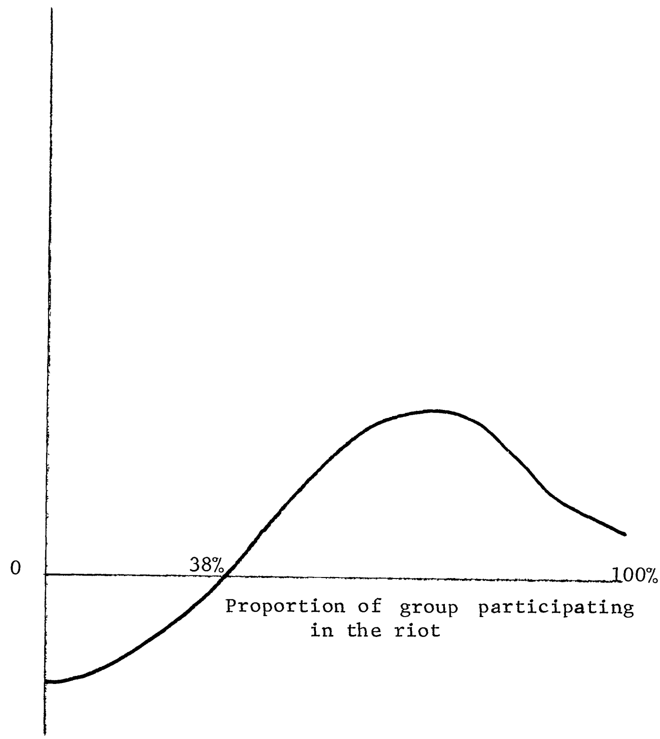

FIG. 3.—Net benefit to an individual, with threshold 38%, of joining a riot, plotted against the proportion of the group participating. (Total benefits minus total costs.)

Measurement and Verification

Because thresholds are behavioral dispositions, they are difficult to measure with confidence before the behavior actually occurs. The situation is similar to that in microeconomics, where the theory is built on the notion that each consumer or producer has a demand or supply schedule—a quantity of the commodity which he will buy or produce at any conceivable price. (Actors in threshold models have, correspondingly, a behavior they will enact for any possible distribution of others' decisions. In both cases, the schedules derive from underlying utility functions.) Since most possible prices are never observed, and economists rarely study the schedules of particular individuals directly, there is normally little direct evidence about these curves which provide the basis for microeconomic theory.

The two main factors which prevent this from causing impossible difficulties in economics are similar to factors operating for threshold models as well. First, much information about supply and demand schedules can be inferred from consumer characteristics, product substitutability, and aggregate outcomes, without direct measurement. Second, the most important thing to know about these schedules is not their exact details, but the level of change in prices to be expected when economic conditions change and the range over which the equilibrium price implied by the curves can be expected to be stable.

For threshold models, we can attempt to index an individual's threshold by the exact number of others who have made a decision before he does. Consider, for example, the adoption of an innovation in some rural village. A complete threshold distribution can be inferred from the exact time sequence of adoptions. This tautological procedure would not afford much explanatory power but could at least allow us to investigate the relation between thresholds and background characteristics, attitudes, values, and social positions. One could then regress thresholds on these various independent variables and use the resulting equations to predict threshold distributions—and hence outcomes—in villages not yet exposed to the innovation.[11]

Such a procedure carries at least two important potential sources of error: (1) Thresholds may be only imperfectly measured by one's position in the time sequence of adoption; measurement error, imperfect information, and chance personal events unrelated to the innovation may result in this. (2) Where the R^2 of multiple regressions falls much under 1.0, the predicted threshold distributions may vary significantly from the true ones. Villages whose distributions have highly stable equilibria might nevertheless yield good predictions from this procedure.

[11] I am indebted to Michael Hannan for this suggestion.

After-the-fact distributions also give us the opportunity to see what kinds of situations characterize villages which had successful versus unsuccessful levels of adoption. Of interest would be whether the results were largely impervious to change or whether small differences might have generated much higher or lower levels of success—as would be the case if distributions had unstable equilibria. Either would have important policy implications.

Dozier (1977) tested threshold models with data from 23 rural Korean villages which gave, for each woman, the month, if any, of first adoption of family planning practices over the period 1964–73. (These data are described more fully in Park et al. 1974.) He split the sample into 18 villages from which he computed multiple-regression equations for thresholds of individual women and five villages for which he used these equations to predict threshold distributions and thence, by forward recursion, outcomes. He then compared predictions obtained in this way with predictions obtained from simple-regression methods without threshold models, comparing both to the true outcomes.

Results suggest that when the multiple regression explains less than 50% of the variance in observed thresholds predictions are worse than those obtained from the standard methods. But as the variance explained edges over 50%, "the accuracy of the recursion model improves dramatically" and it predicts far better than the alternatives (Dozier 1977, p. 230). Many such predictions approach 100% accuracy in these data, which would be a powerful result from a model which predicts only 50% of the variance in thresholds. Caution is necessary, however, since some of the variables in the regression equations could not have been measured before the time period covered by the data and are highly correlated with thresholds in ways which deprive them of true explanatory power (see Dozier 1977, chap. 8).

While much more needs to be said about the problems of empirical application of these models, this is not the main intent of the present paper, which means only to summarize the theoretical development to date.

SUMMARY

This paper presents models of collective behavior, based on behavioral thresholds, which account for collective outcomes by simple principles of aggregation. The models are particularly valuable in helping to understand situations where outcomes do not seem intuitively consistent with the underlying individual preferences. Such "paradoxes" may occur far more than we realize, since we observe mainly outcomes and tend to assume that the preferences generating them were consistent with rather than opposed or unrelated to them. Our reluctance to recognize paradox is consistent with an everyday social construction of reality which construes social systems as smoothly operating entities, without discontinuities or incomprehensible events (see Berger and Luckmann 1968).

By explaining paradoxical outcomes as the result of aggregation processes, threshold models take the "strangeness" often associated with collective behavior out of the heads of actors and put it into the dynamics of situations. Such models may be useful in small-group settings as well as those with large numbers of actors. Their greatest promise lies in analysis of situations where many actors behave in ways contingent on one another, where there are few institutionalized precedents and little preexisting structure. These situations are central in social life but not an important focus of theoretical analysis either in micro- or macrosociology. Providing tools for analyzing them is part of the important task of linking micro to macro levels of sociological theory.

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