Modeling the Relationship between Links and Communities for Overlapping Community Detection

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Outline

- Introduction and Motivation
- Background Study
  - Community Detection
  - Overlapping Community Detection (OCD)
  - Matrix Factorization (MF) framework for OCD
- Research Work
  - PNMF: Preference-based Non-negative MF Model
  - LNMF: Locality-based Non-negative MF Model
  - MD-NMF: Mutual Density-based Non-negative MF Model
  - HNMF: Homophily-based Non-negative MF Model
- Conclusion

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Introduction

Big data era is coming!

http://mazakali.com/cannabis-beyond-information-era/

INTRODUCTION

https://blog.unbelievable-machine.com/en/what-is-big-data-definition-five-vs

Introduction

NETWORK DATA

INTRODUCTION

INTRODUCTION

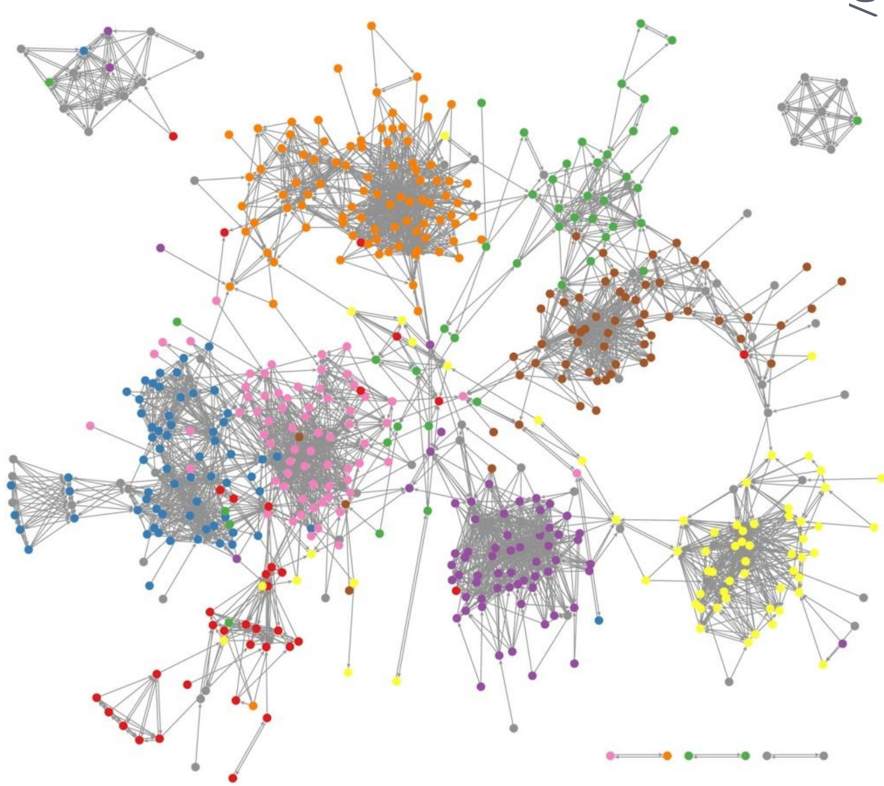
- facebook statistics 7/26/17
  - Over 2.01 billion monthly active Facebook users (Facebook MAUs) which is a 17% increase year over year.
  - 1.32 billion people on average who log onto Facebook daily active users (Facebook DAU), which represents a 17% increase year over year.
  - Age 25 to 34, at 29.7% of users, is the most common age demographic.

https://zephoria.com/top-15-valuable-facebook-statistics/

INTRODUCTION

- "It is a matter of common experience that communities exist in networks … Although not precisely defined, communities are usually thought of as sets of nodes with better connections amongst its members than with the rest of the world."

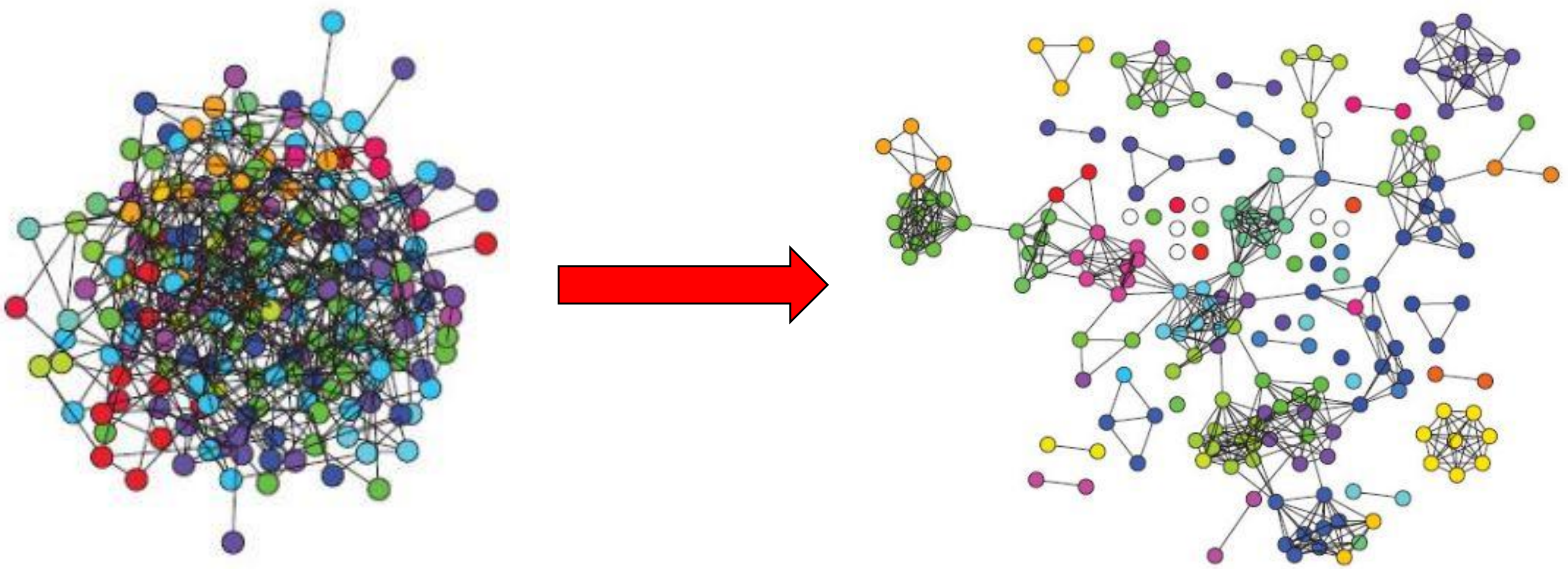
Community Structure in Large Social and Information
Michael W. Mahoney, MMDS 2008



Caltech Facebook network
http://sociograph.blogspot.hk/2013/05/revealing-community-structure-with.html

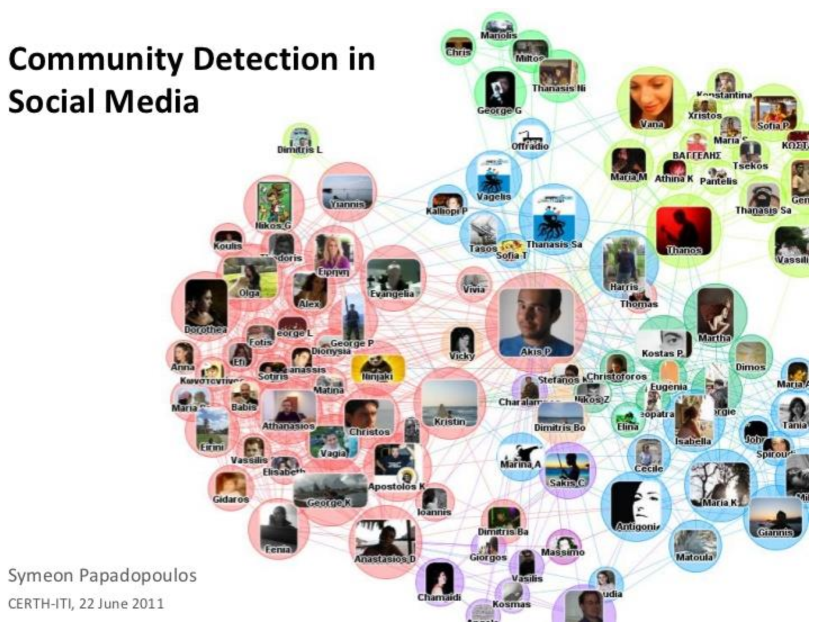
INTRODUCTION

- Community Detection: uncovering densely-connected small components in large networks

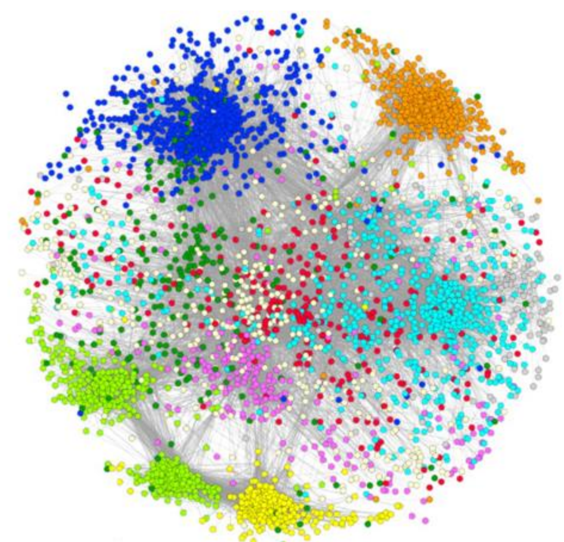


INTRODUCTION

- Applications:
  - complex graph visualization
  - group discovery in social networks
  - functional unit tracking in protein networks, etc.

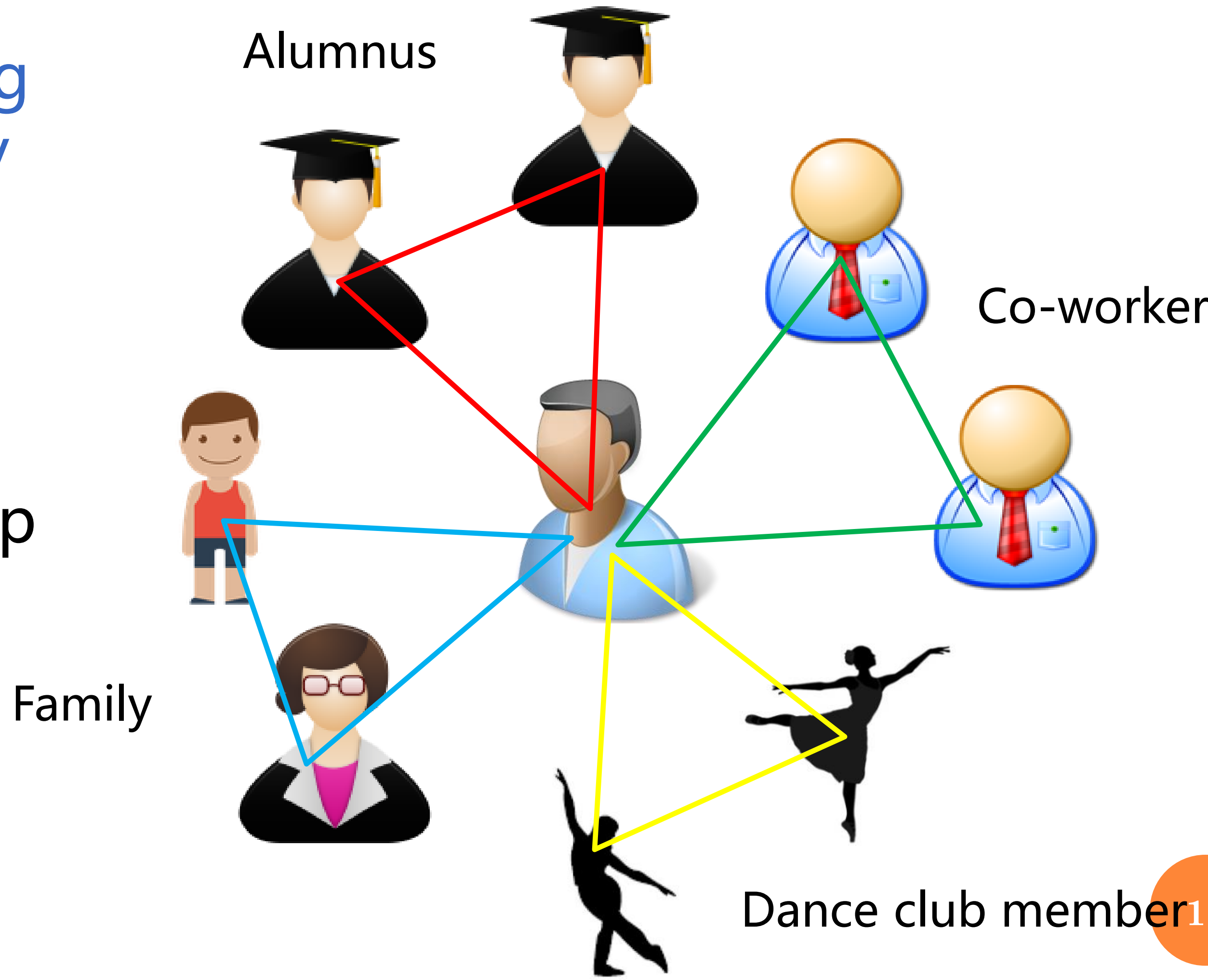


protein-protein interaction network

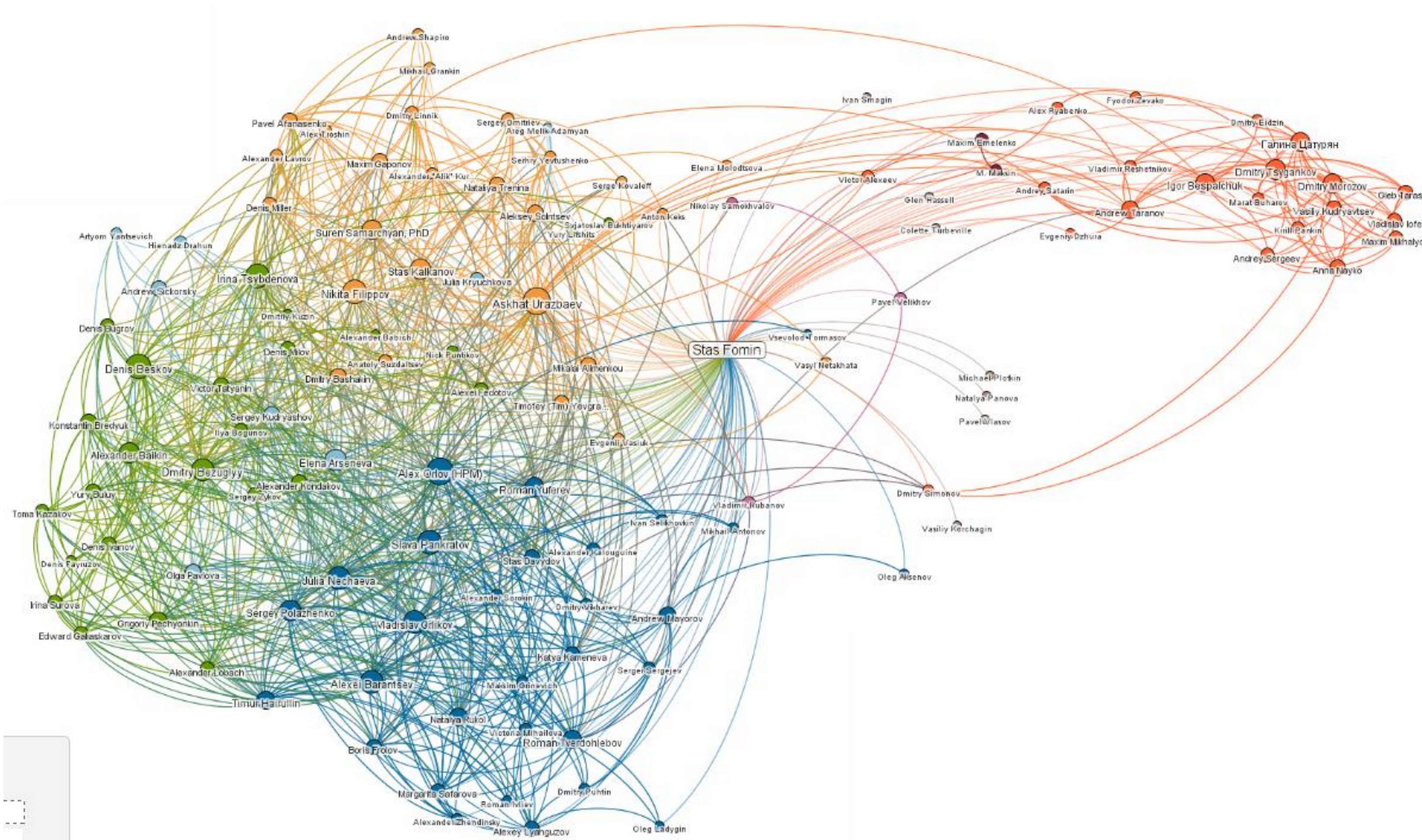


INTRODUCTION

- Overlapping Community Detection: allowing multiple community membership



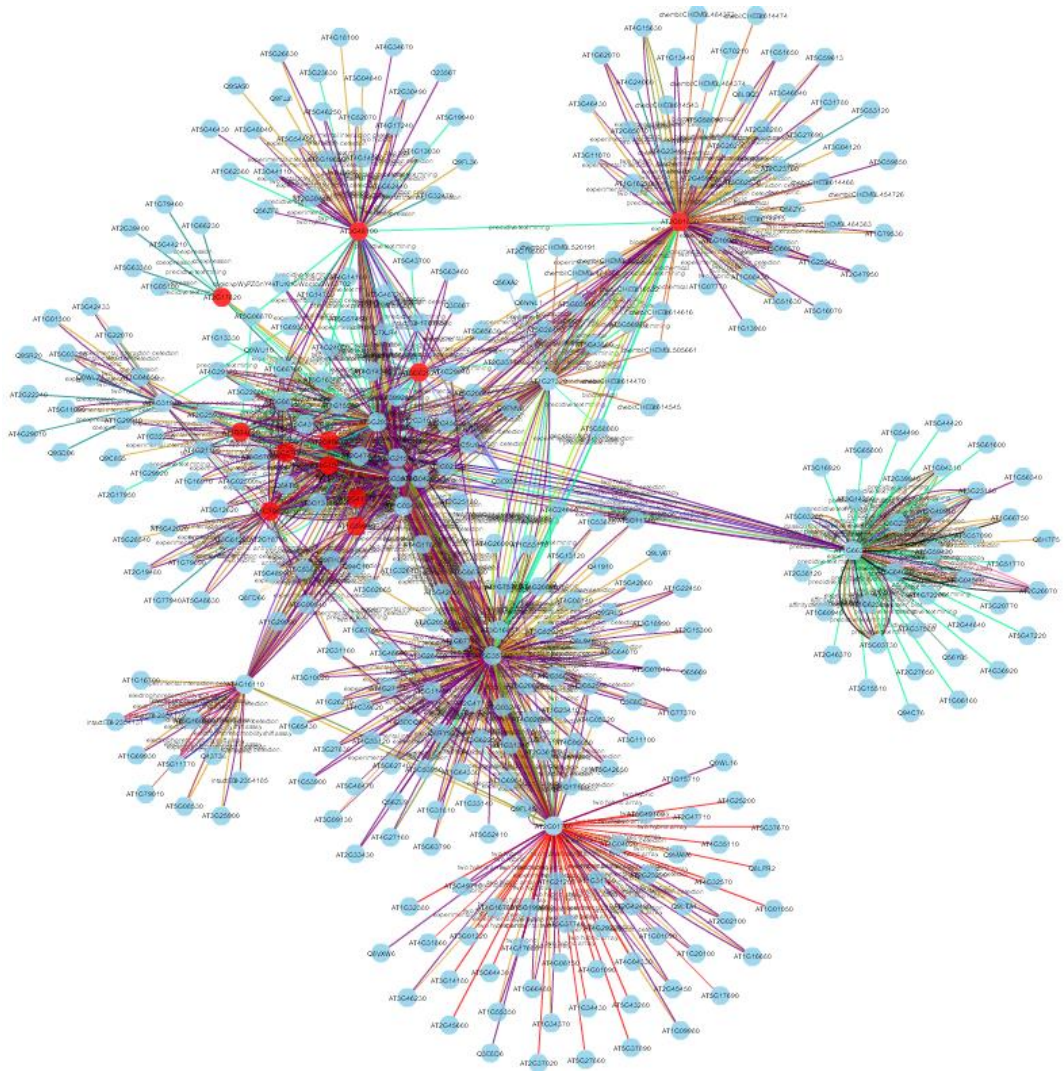
INTRODUCTION



A LinkedIn connection network

http://discopal.ispras.ru/img_auth.php/f/f4/StasFomin-Linkedin.png

INTRODUCTION

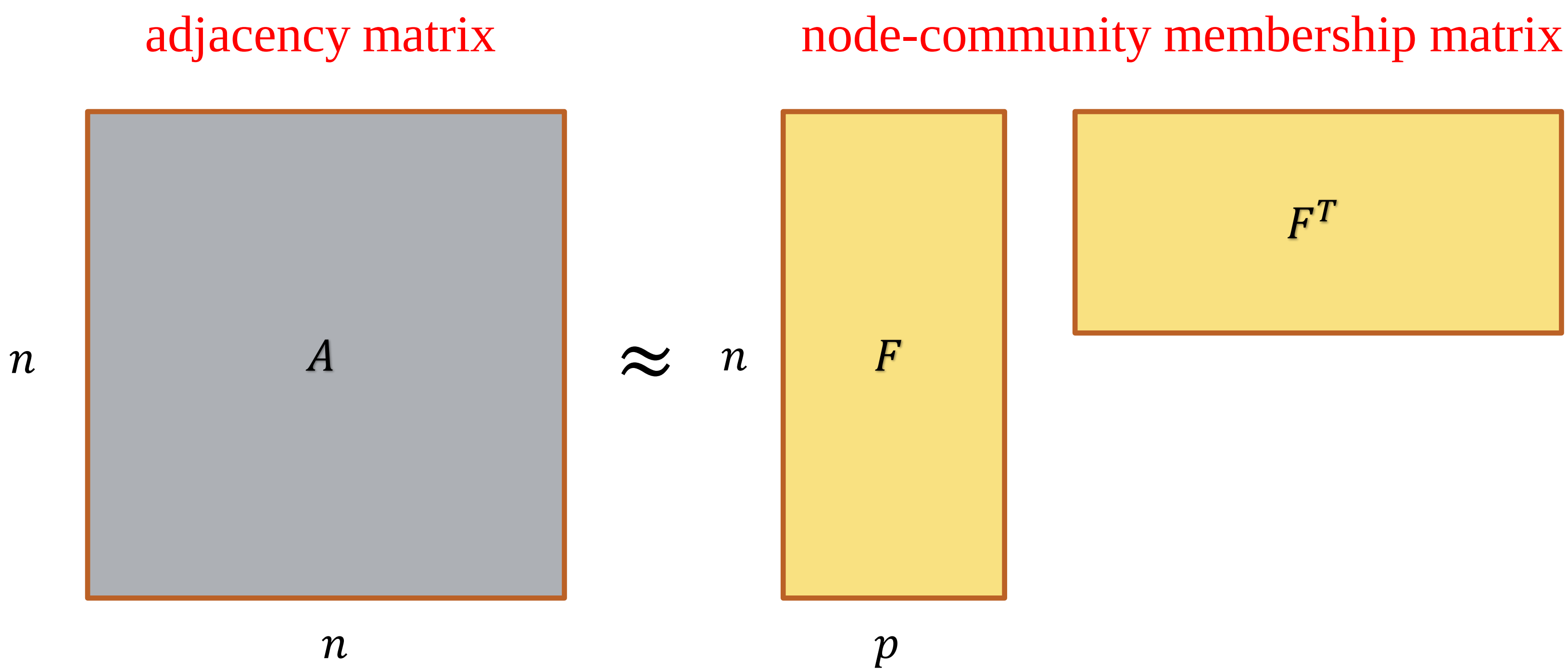


A gene network

http://gmdd.shgmo.org/Computational-Biology/ANAP/ANAP_V1.1/help/anap-userguide/figures/anap9.png

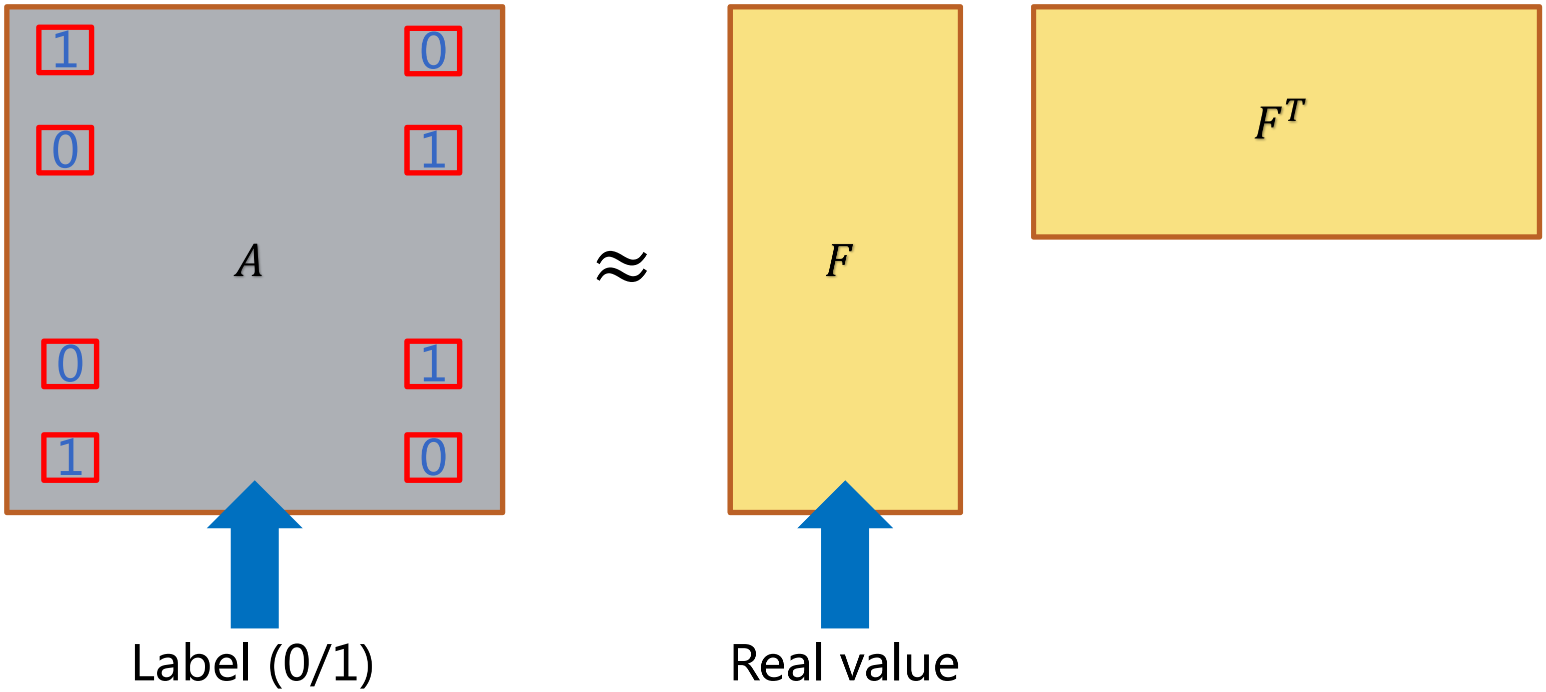
Introduction

Matrix Factorization (MF) Framework



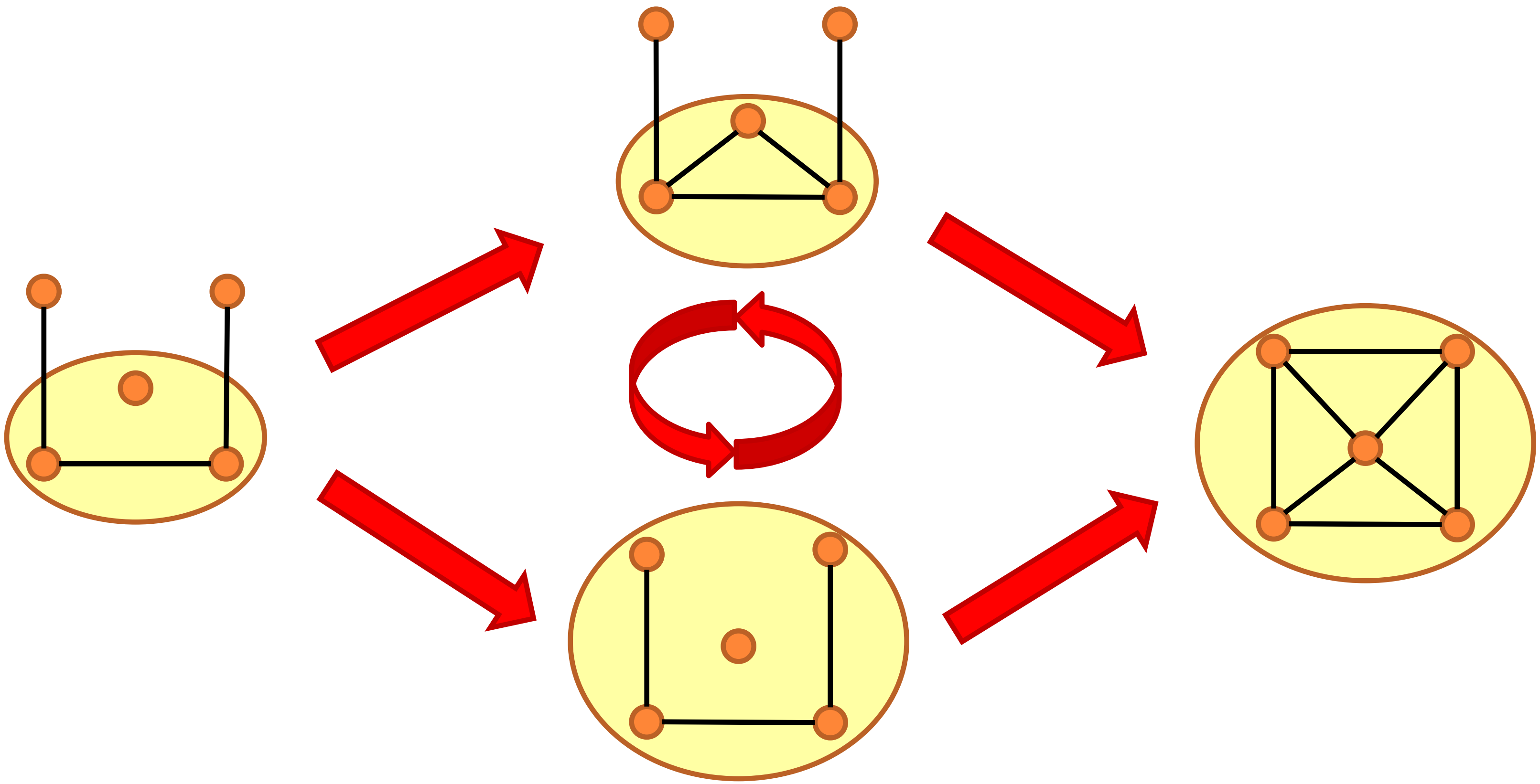
- An objective function is used to learn parameters in F.

Motivation



- A typical objective function is to minimize $||A - FF^T||_F$.
- Point-wise approximation is problematic!

MOTIVATION



- Community is a result of mutual enhancement between links and community itself.

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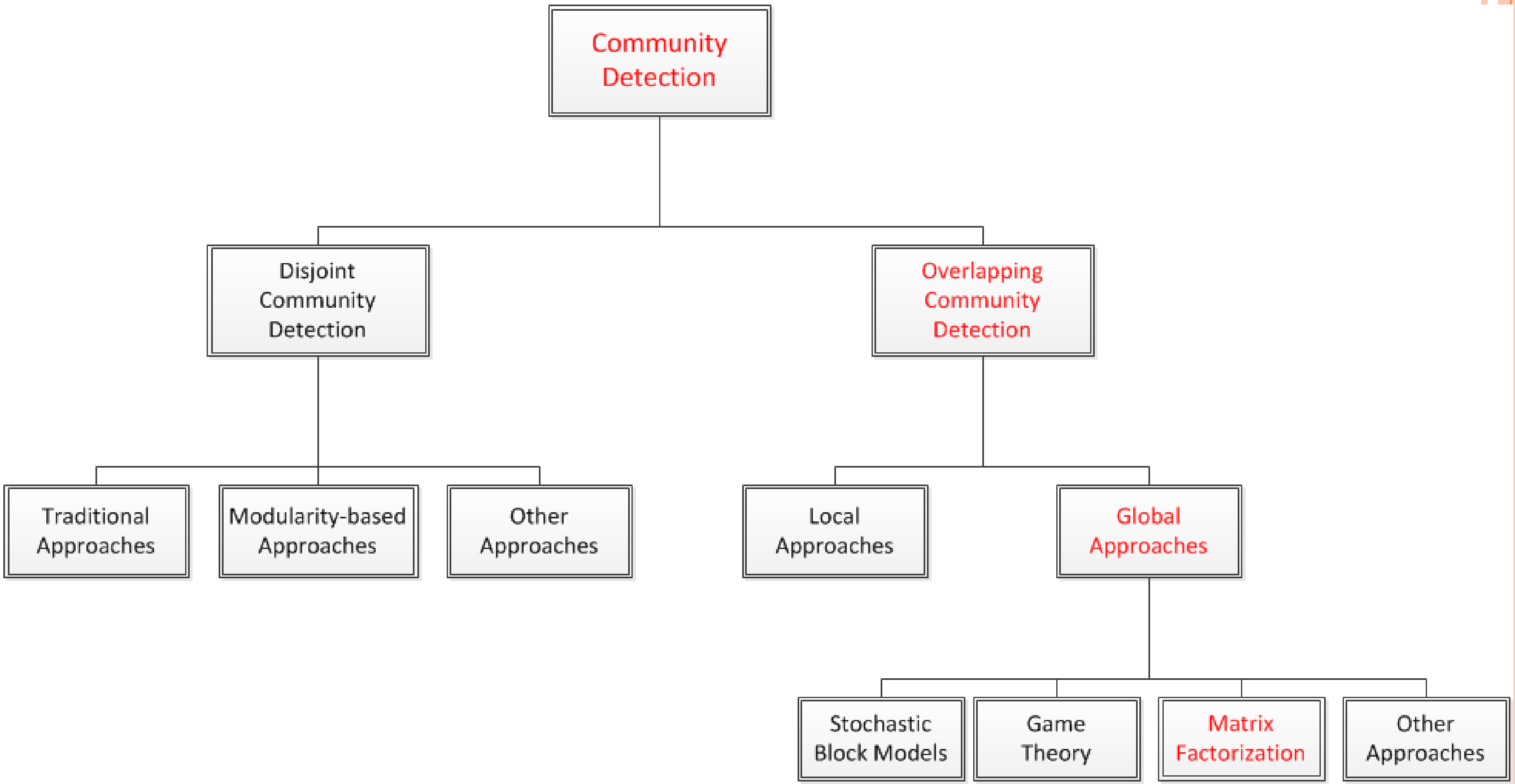
Background Study



Community Detection

- A graph or network $G(V,E)$ with adjacency matrix A

- Definition 1 (Community): A community C is a subset of V which consists of all nodes with a certain feature.

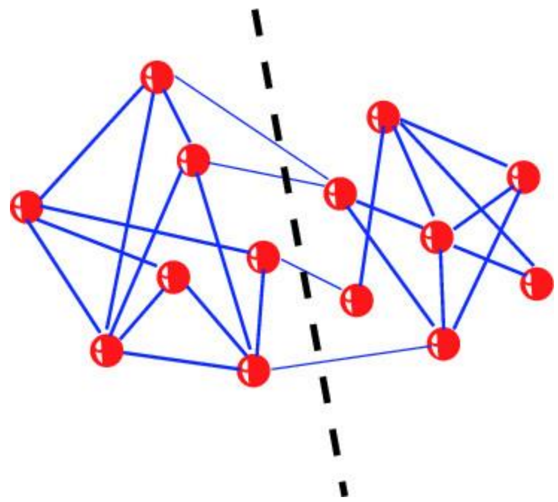
- Definition 2 (Community Detection): Community detection aims to find a set of communities $S=\{C_i|C_i \neq \emptyset, C_i \neq C_j, 1 \leq i < j \leq p\}$, which minimizes an objective function f, i.e.,

$$\underset{S}{argmin}\, f(G,S),$$

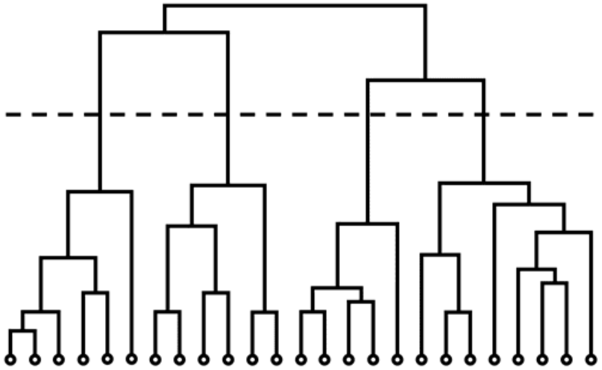
where p is the number of communities.

Community Detection

- Graph partitioning algorithm
  - cut size is the number of edges across two groups
  - weakness: the number and size of groups need to be determined beforehand



- Hierarchical clustering
  - agglomerative or divisive
  - a similarity metric is required
  - weakness: hard to decide when to stop



Community Detection

- Modularity [Newman and Girvan 2004] is the most popular quality function to measure how good the detected communities are.

$$Q = \frac{1}{2|E|} \sum_{u,v \in V} \left(A_{u,v} - \frac{d(u)d(v)}{2|E|} \right) I_{u,v}$$

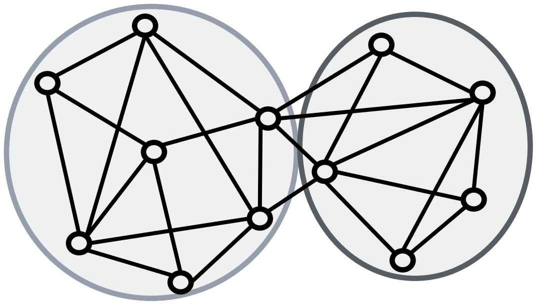
null model

indicator function

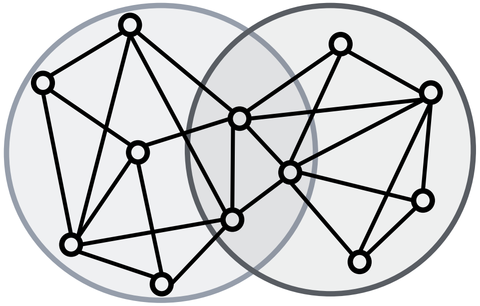
- Modularity can also be used as an optimization objective. Modularity optimization is by far the most widely-used algorithms for classical community detection.

Overlapping Community Detection (OCD)

- Disjoint community detection has two constrains:
  - Exhaustive - $C_1 \cup \cdots \cup C_p = V$;
  - Disjoint - $C_i \cap C_j = \emptyset$ for any $i \neq j$.



- Overlapping community detection has no such constraints.
  - Advantage: more common in real-world networks
  - Challenge: classic algorithms are no longer feasible



Overlapping Community Detection (OCD)

- Local approaches
  - Divide-and-conquer
    - k-clique searching [Palla et al. 2005; Kumpula et al. 2008]
    - Seed set expansion [Whang et al. 2013]
  - Modified disjoint community detection algorithms
    - Link clustering [Ahn et al. 2010]
    - Label propagation [Coscia et al. 2012]

- Global approaches
  - Stochastic block models [Airoldi et al. 2008]
  - Game theory framework [Chen et al. 2010]
  - Matrix factorization

Matrix Factorization Framework for OCD

- Definition 3 (Overlapping Community Detection via Matrix Factorization): Given a graph $G(V, E)$ with its adjacency matrix $A \in \{0,1\}^{n \times n}$, the objective of overlapping community detection via matrix factorization is to find a number of matrices $P_1, P_2, \ldots, P_k$ so that the product of these matrices $P_1 P_2 \ldots P_k$ can minimize a loss function l, i.e.,

$$\underset{F}{argmin}\, l(A, P_1 P_2 \ldots P_k),$$

where n is the number of nodes, p is the number of communities and C is the set of communities.

- Variant: factorization form, loss function

Matrix Factorization Framework for OCD

- [Psorakis et al. 2011]

$$l(A, W, H) = p(A|W, H)$$

- [Wang et al. 2011]

$$l(A, F) = \left|\left|A - FF^T\right|\right|_F^2$$

- [Zhang and Yeung 2012]

$$l(A, F, B) = \left|\left|A - FBF^T\right|\right|_F^2 \quad \text{squared loss}$$

$$l(A, F, B) = KL(A, FBF^T) \quad \text{KL-divergence}$$

- [Yang and Leskovec 2013]

$$l(A, F) = \sum_{(u,v)\in E} p(u, v) + \sum_{(u,v)\notin E} (1 - p(u, v))$$

$$p(u, v) = e^{1 - F_u F_v^T}$$

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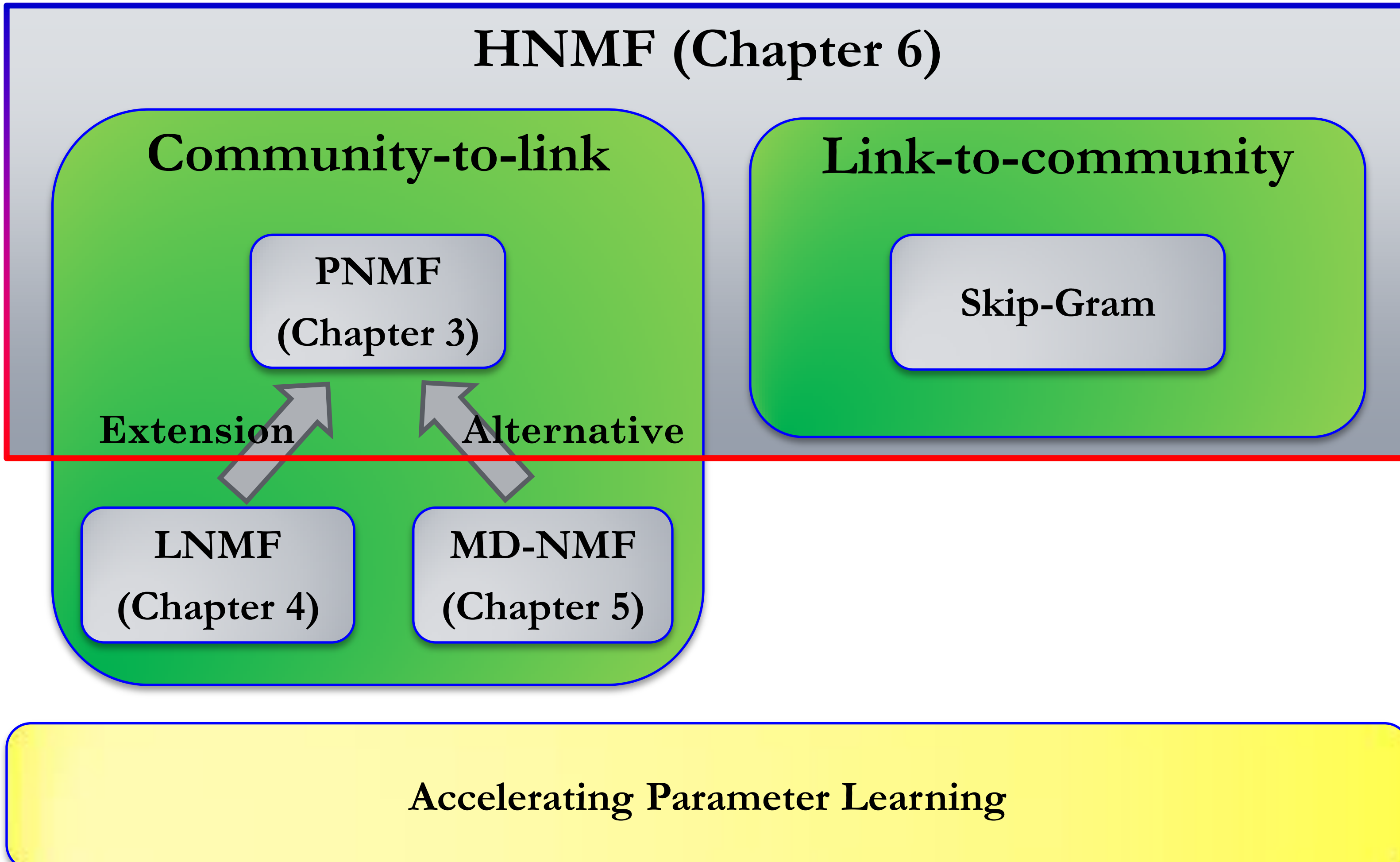
RESEARCH WORK - HIERARCHY



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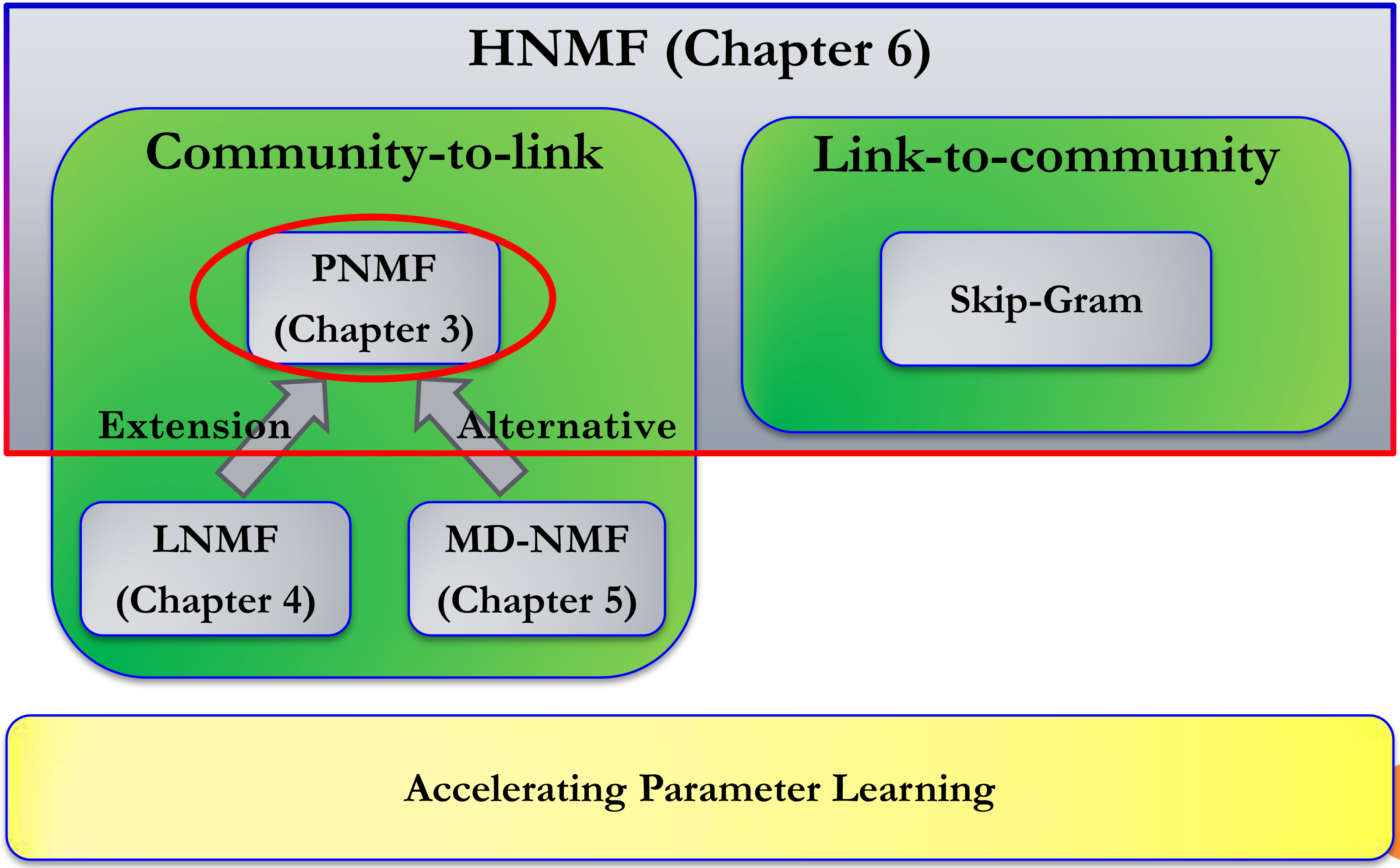
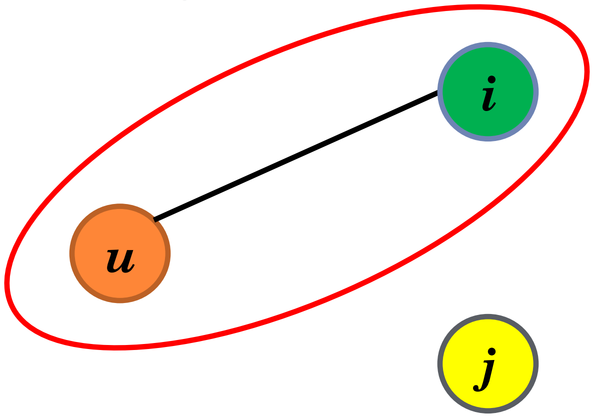


PNMF - ASSUMPTION



- Preference - the tendency to build links. Preference on neighbors is larger than preference on non-neighbors, i.e., $r_{u,i} > r_{u,j}$.
- Assumption: if u share more communities with i than j, u has a higher preference on i compared to j.

PNMF - FORMULATION

- The objective function – maximizing the sum of log-likelihood to be consistent with the original network:

$$\sum_{u \in V} \log \mathcal{P}(>_u) = \sum_{u \in V} \sum_{i \in N^+(u), j \in N^-(u)} \log \mathcal{P}(r_{u,i} > r_{u,j})$$

$N^+(u)$: set of u' s neighbors, $N^-(u)$: set of u' s non-neighbors.

- $\mathcal{P}(r_{u,i} > r_{u,j}) = \sigma(F_u F_i^T - F_u F_j^T)$, where $\sigma(x) = \frac{1}{1+e^{-x}}$ is the sigmoid function, F_i is the i-th row of node-communitiy weight matrix F.

- Adding $||F||_F^2$ as a regularization term to prevent overfitting.

PNMF - EXPERIMENTS

ground-truth node link

Dataset	GT	V	E
Dolphins	N	62	159
Les Misérables	N	77	254
Books about US politics	N	105	441
Word adjacencies	N	112	425
American college football	N	115	613
Jazz musicians	N	198	2,742
Network science	N	1,589	2,742
Power grid	N	4,941	6,594
High-energy theory	N	8,361	15,751
DBLP	Y	317,080	1,049,866
Amazon	Y	334,863	925,872
YouTube	Y	1,134,890	2,987,624

- Data source
  - http://www-personal.umich.edu/mejn/netdata
  - http://snap.stanford.edu/data/

PNMF - EXPERIMENTS

- Metrics
  - Overlapping Modularity (M)

$$M = \frac{1}{2|E|} \sum_{u,v \in V} \left(A_{u,v} - \frac{d(u)d(v)}{2|E|} \right) |C_u \cap C_v|$$

  - F_1 score
    - harmonic mean of precision and recall of detected communities (S) from ground-truth (S')

$$precision(S_i) = \max_j \frac{|S'_j \cap S_i|}{|S_i|}$$

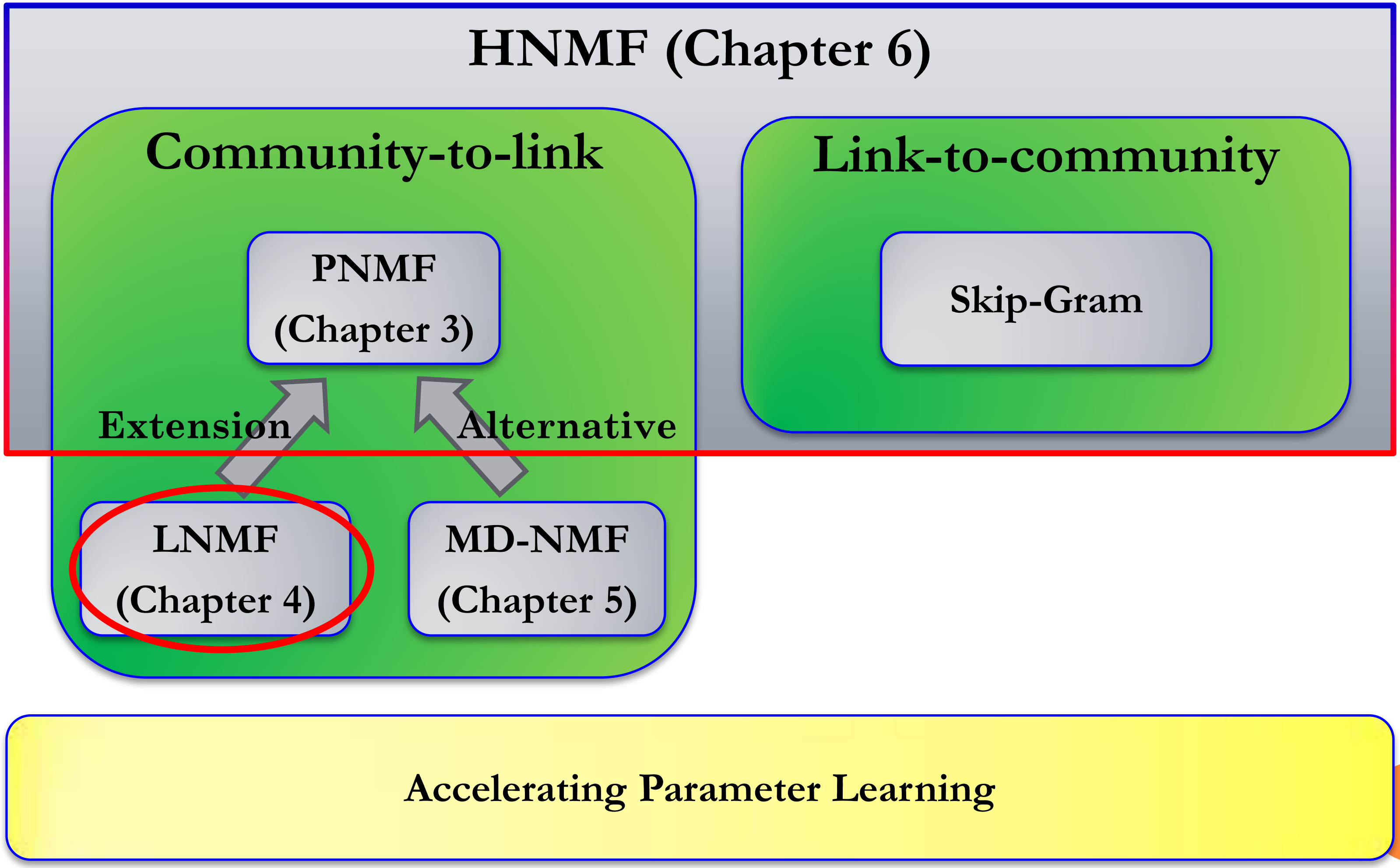
$$recall(S_i) = \max_j \frac{|S'_j \cap S_i|}{|S'_j|}$$

PNMF - EXPERIMENTS

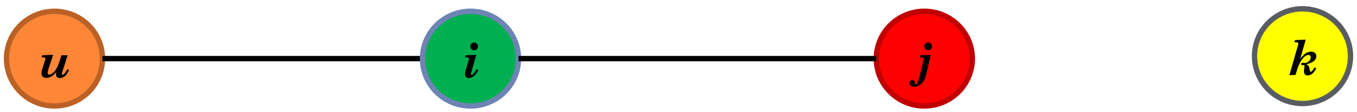
Dataset	Metric	SCP	LC	BNMF	BNMTF	BigCLAM	PNMF
Dolphins	M	0.3049	0.6538	0.5067	0.5067	0.4226	**0.9787**
Les Misérables	M	0.3066	0.7730	0.1247	0.1031	0.5395	**1.1028**
Books about US politics	M	0.4955	0.8507	0.4613	0.4924	0.5290	**0.8640**
Word adjacencies	M	0.0707	0.2705	0.2539	0.2677	0.2312	**0.6680**
American college football	M	0.6050	0.8907	0.5584	0.5733	0.5175	**1.0492**
Jazz musicians	M	0.0114	1.1424	0.1133	0.1118	**1.1438**	0.9357
Network science	M	0.7286	0.9558	0.6607	0.7413	0.5026	**1.6570**
Power grid	M	0.0439	0.3713	0.3417	0.3682	1.0097	**1.1051**
High-energy theory	M	0.5427	**0.9965**	0.5648	0.6004	0.9636	0.9725
DBLP	F_1	0.0967	0.0402	-	-	0.0390	**0.0985**
Amazon	F_1	0.0315	0.0070	-	-	**0.0441**	0.0419
YouTube	F_1	0.0445	-	-	-	0.0194	**0.0605**

- Baseline
  - SCP – Sequential Clique Percolation [Palla et al. 2005; Kumpula et al. 2008]
  - LC – Link Clustering [Ahn et al. 2010]
  - BNMF – Bayesian Non-negative Matrix Factorization [Wang et al. 2011]
  - BNMTF – Bounded Non-negative Matrix Tri-Factorization [Zhang and Yeung 2012]
  - BigCLAM – Cluster Affiliation Model for Big Networks [Yang and Leskovec 2013]

Research Work - Hierarchy



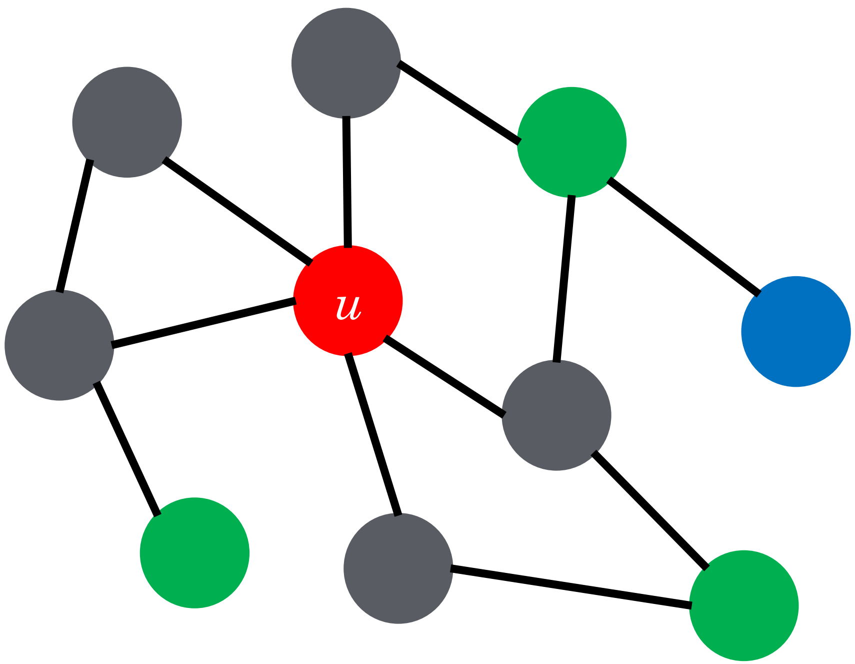
LNMF - Motivation



- Locality: A friend of your friend (but not your direct friend) is more likely to share communities with you than a total stranger, i.e., $r_{u,i} > r_{u,j} > r_{u,k}$.
- An extension of the preference system of PNMF.

LNMF - Assumption

k-degree local network ($k = 2$)



- u source node
- neighbor
- local non-neighbor
- distant non-neighbor

$$r_{u,neighbor} > r_{u,local\ non-neighbor} > r_{u,distant\ non-neighbor}$$

LNMF – Formulation

- For node u, The objective function – maximizing the sum of log-likelihood to be consistent with the original network:

$$\log \mathcal{P}(>_u) = \sum_{i \in N^+(u), j \in LN(u)} \log \mathcal{P}(r_{u,i} > r_{u,j}) + \sum_{j \in LN(u), d \in DN(u)} \log \mathcal{P}(r_{u,j} > r_{u,d})$$

  $N^+(u)$: set of u' s neighbors, $LN(u)$: set of u' s local non-neighbors, $DN(u)$: set of u' s distant non-neighbor.

- LNMF degrades to PNMF when we do not separate local non-neighbors and distant non-neighbors.

LNMF - Results

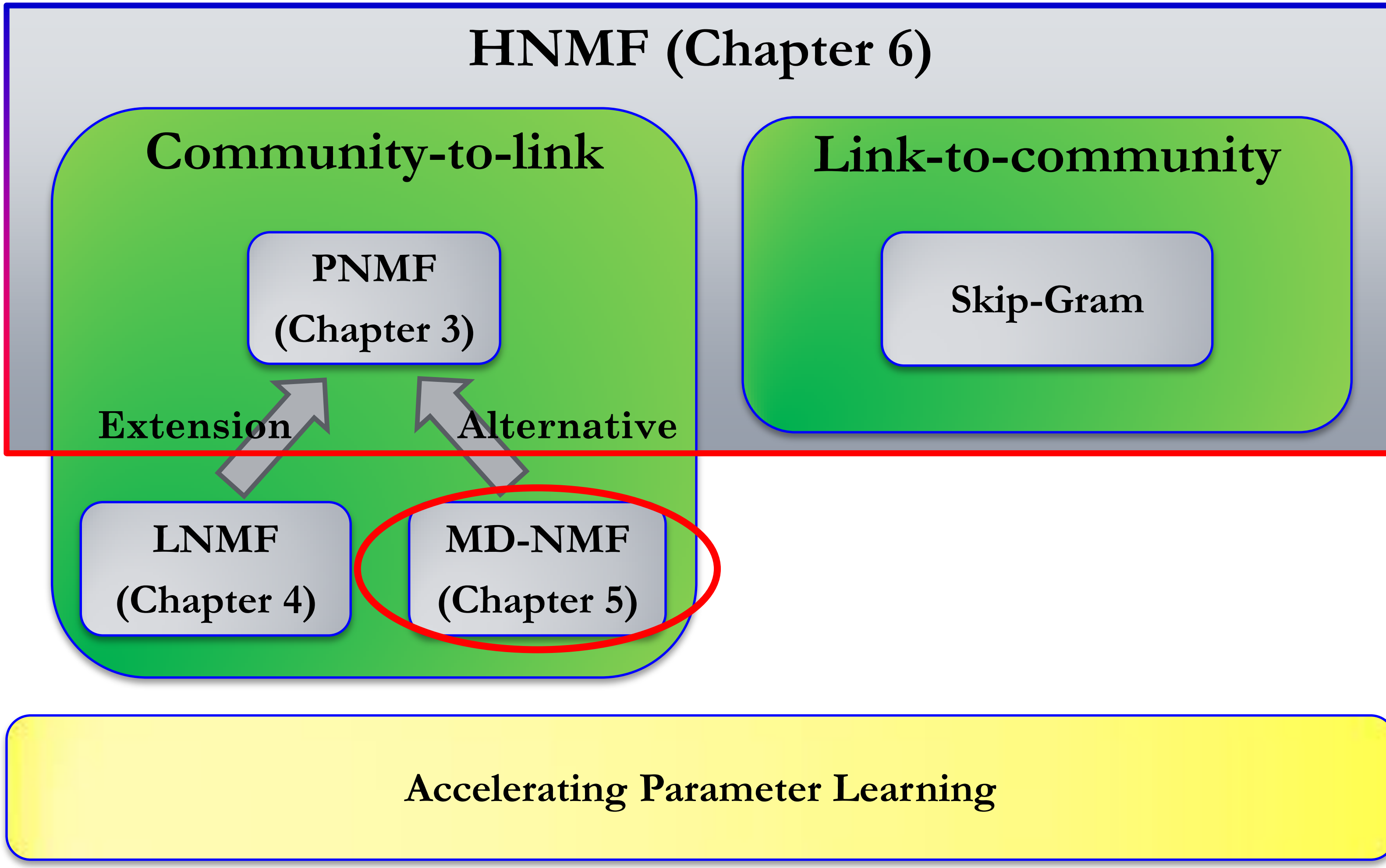
Dataset	SCP	LC	BNMF	BNMTF	BigCLAM	PNMF	LNMF(RI)
Dolphins	0.305	0.654	0.507	0.507	0.423	0.979	**1.086(10.9%)**
Les Misérables	0.307	0.773	0.125	0.103	0.540	1.103	**1.184(7.3%)**
Books about US politics	0.496	0.851	0.461	0.492	0.529	0.864	**1.270(47.0%)**
Word adjacencies	0.071	0.271	0.254	0.268	0.231	0.668	**0.701(4.9%)**
American College football	0.605	0.891	0.558	0.573	0.518	1.049	**1.235(17.7%)**
Coauthorships in network science	0.729	0.956	0.661	0.741	0.503	1.657	**2.310(39.4%)**

Table 4: **Comparison in terms of modularity. RI: Relative Improvement over PNMF.**

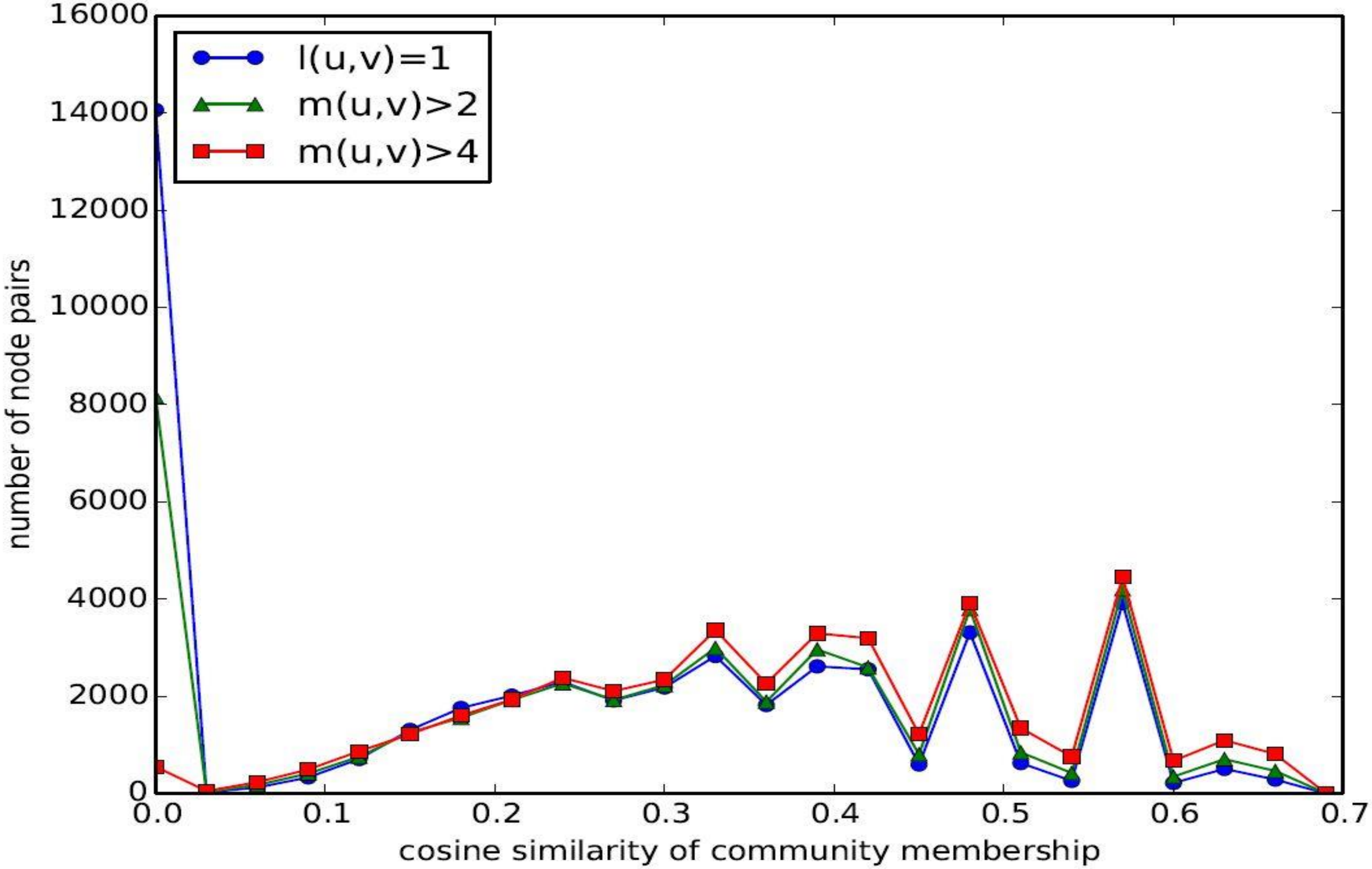
Dataset	BigCLAM	PNMF	LNMF(RI)
DBLP	0.039	0.098	**0.107(9.2%)**
Amazon	0.044	0.042	**0.048(11.4%)**
YouTube	0.019	**0.060**	0.057(0.0%)

Table 5: **Experimental results on SNAP datasets in terms of F_1 score. RI: Relative Improvement over PNMF.**

RESEARCH WORK - HIERARCHY

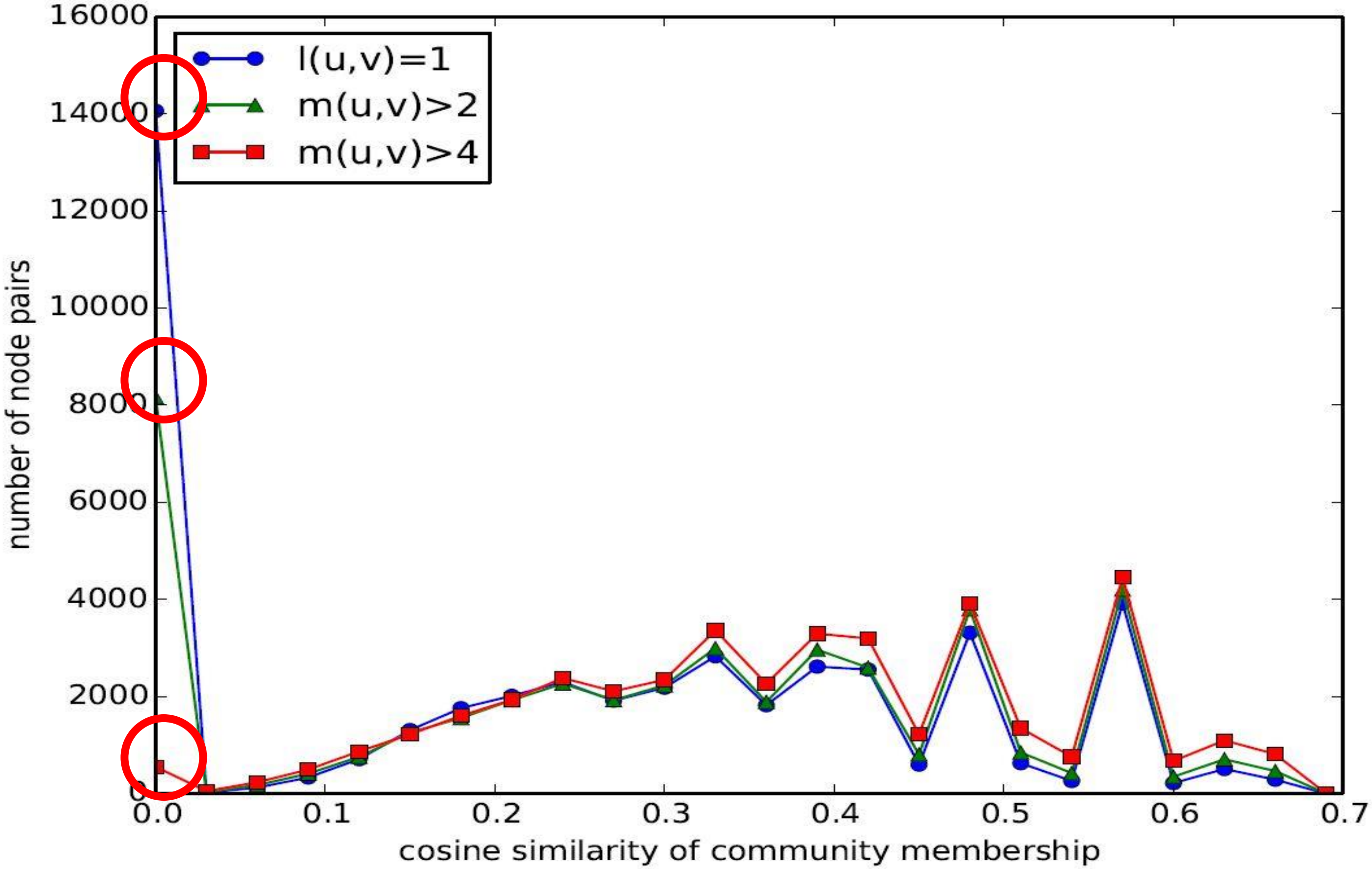


MD-NMF - Motivation



- Mutual Friend is a better indicator than link to reflect the similarity of community membership between two nodes.

MD-NMF - MOTIVATION



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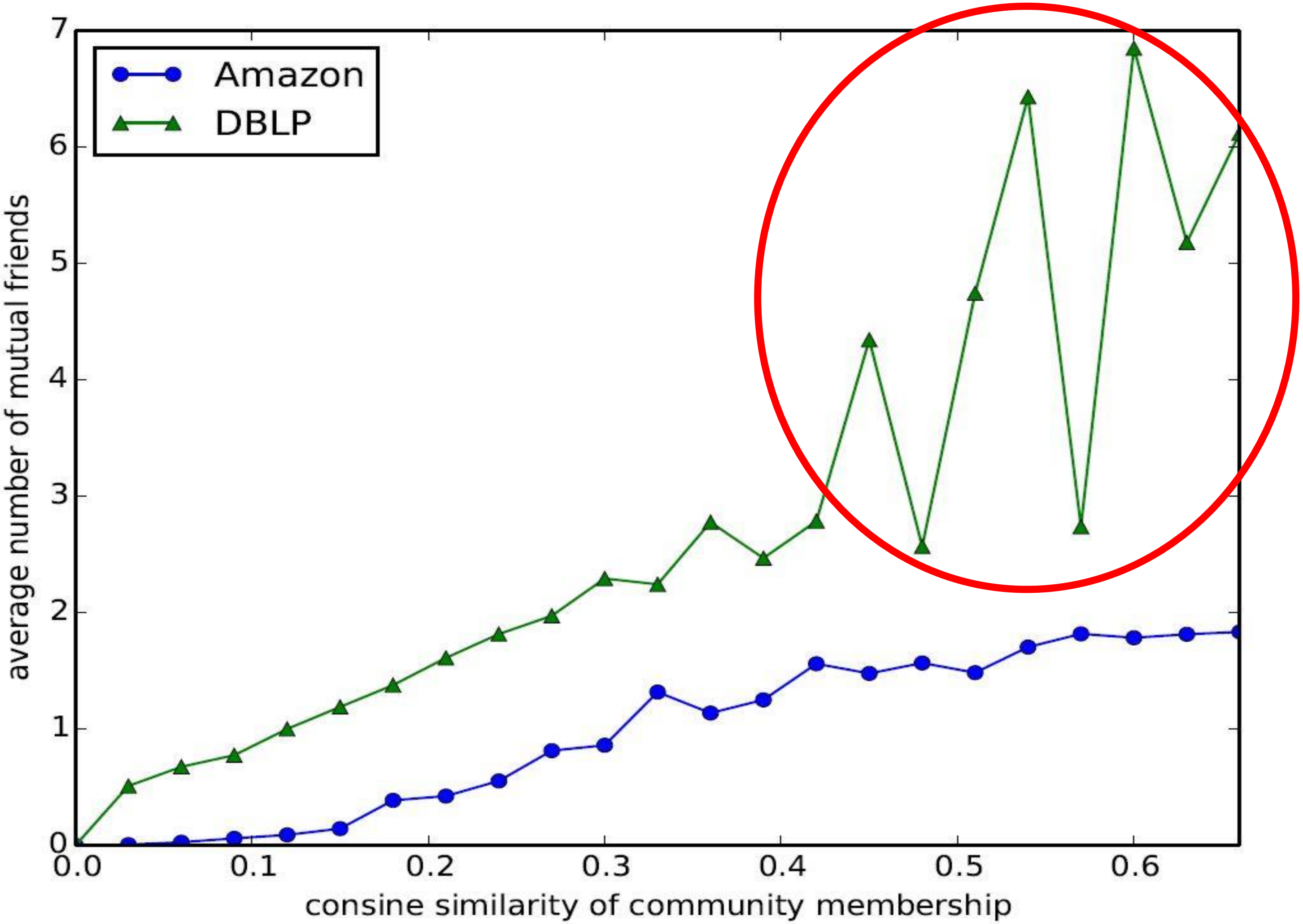
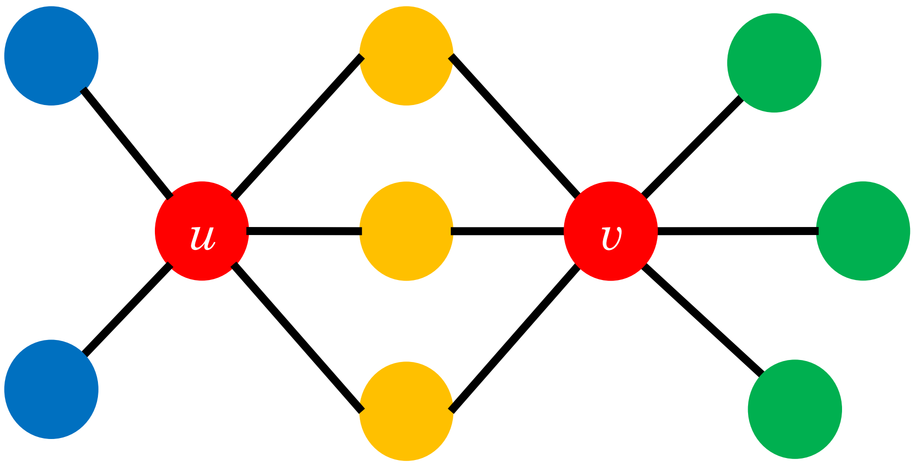
MD-NMF - Motivation

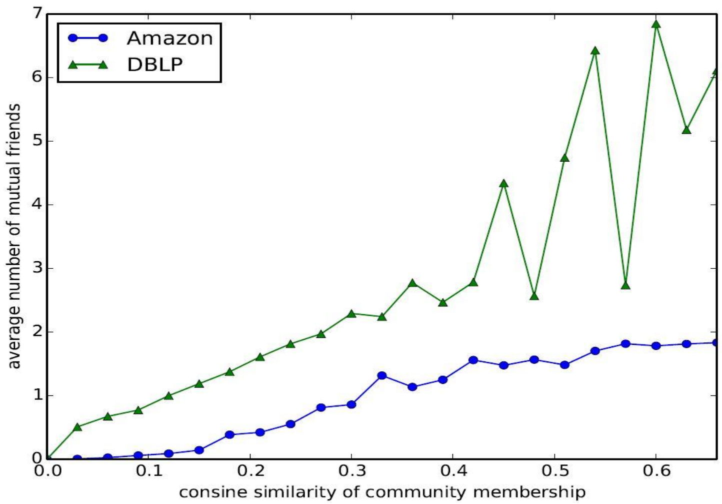


MD-NMF - Motivation

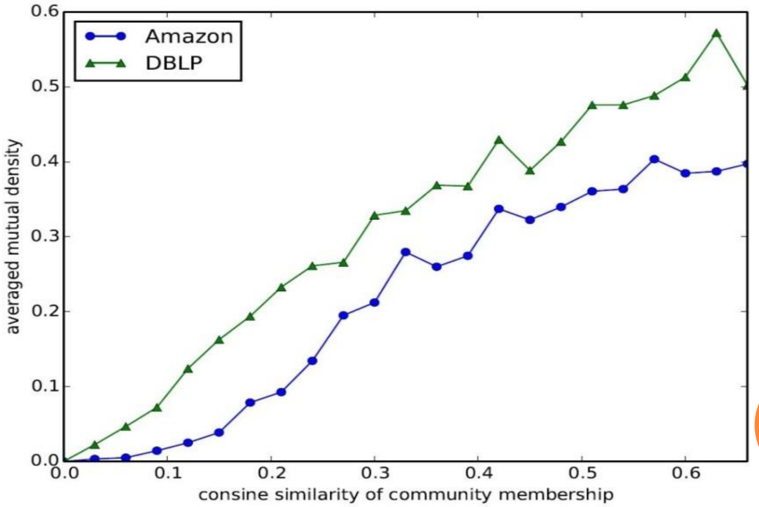
Mutual Density

$$d(u,v) = \frac{|N^+(u) \cap N^+(v)|}{|N^+(u) \cup N^+(v)|}$$





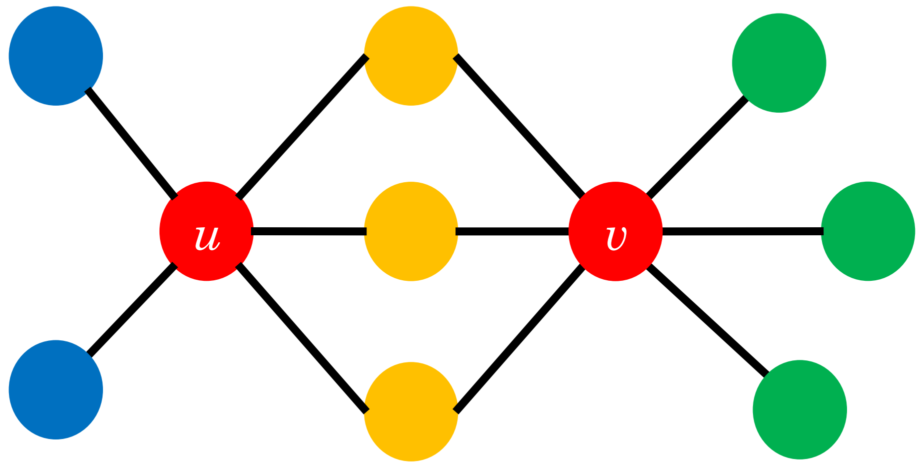
VS

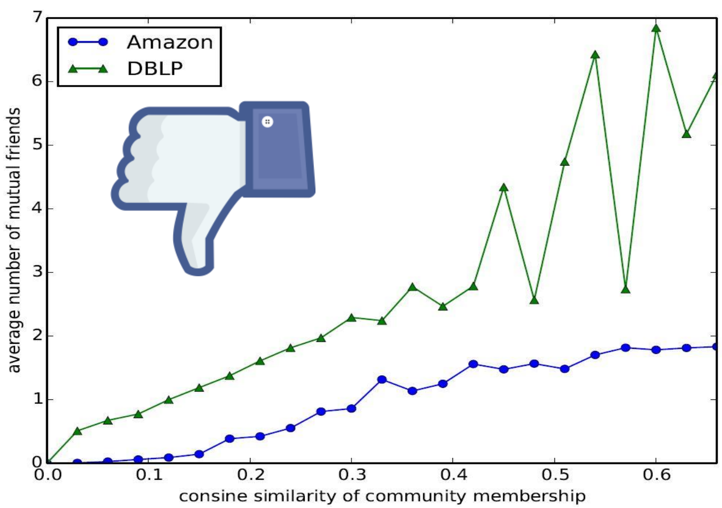


MD-NMF - Motivation

Mutual Density

$$d(u,v) = \frac{|N^+(u) \cap N^+(v)|}{|N^+(u) \cup N^+(v)|}$$





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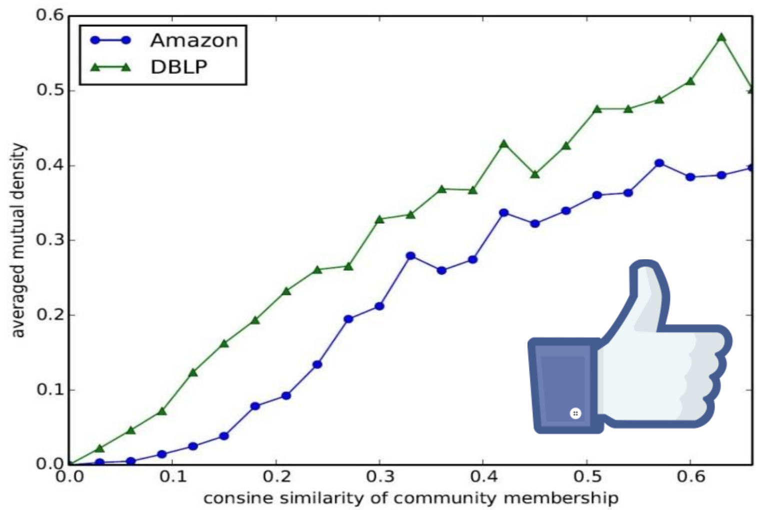


MD-NMF - Formulation

- Set of α-acquaintance: $A(u,\alpha) = \{i | d(u,i) \geq \alpha\}$
- Set of β-stranger: $B(u,\beta) = \{j | d(u,j) \leq \beta\}$
- Assumption: α-acquaintance > β-stranger

- The objective function – maximizing the sum of log-likelihood to be consistent with the original network:

$$\sum_{u \in V} \log \mathcal{P}(>_u | \alpha, \beta) = \sum_{u \in V} \sum_{i \in A(u,\alpha), j \in B(u,\beta)} \log \mathcal{P}(r_{u,i} > r_{u,j})$$

MD-NMF - Results

Dataset	SCP	Demon	BNMF	BNMTF	BigCLAM	PNMF	MD-NMF
Dolphins	0.3049	0.6804	0.5067	0.5067	0.4226	0.9787	**1.0187**
Books about US politics	0.4955	0.4317	0.4613	0.4924	0.5920	0.8640	**0.9871**
American college football	0.6050	0.5402	0.5584	0.5733	0.5175	1.0492	**1.163**
Network Science	0.7286	0.6423	0.6607	0.7413	0.5026	1.6570	**1.6951**
Power grid	0.0439	0.1946	0.3417	0.3682	1.0097	1.1051	**1.228**
High-energy theory	0.5427	0.9621	0.5648	0.6004	0.9636	0.9725	**1.031**

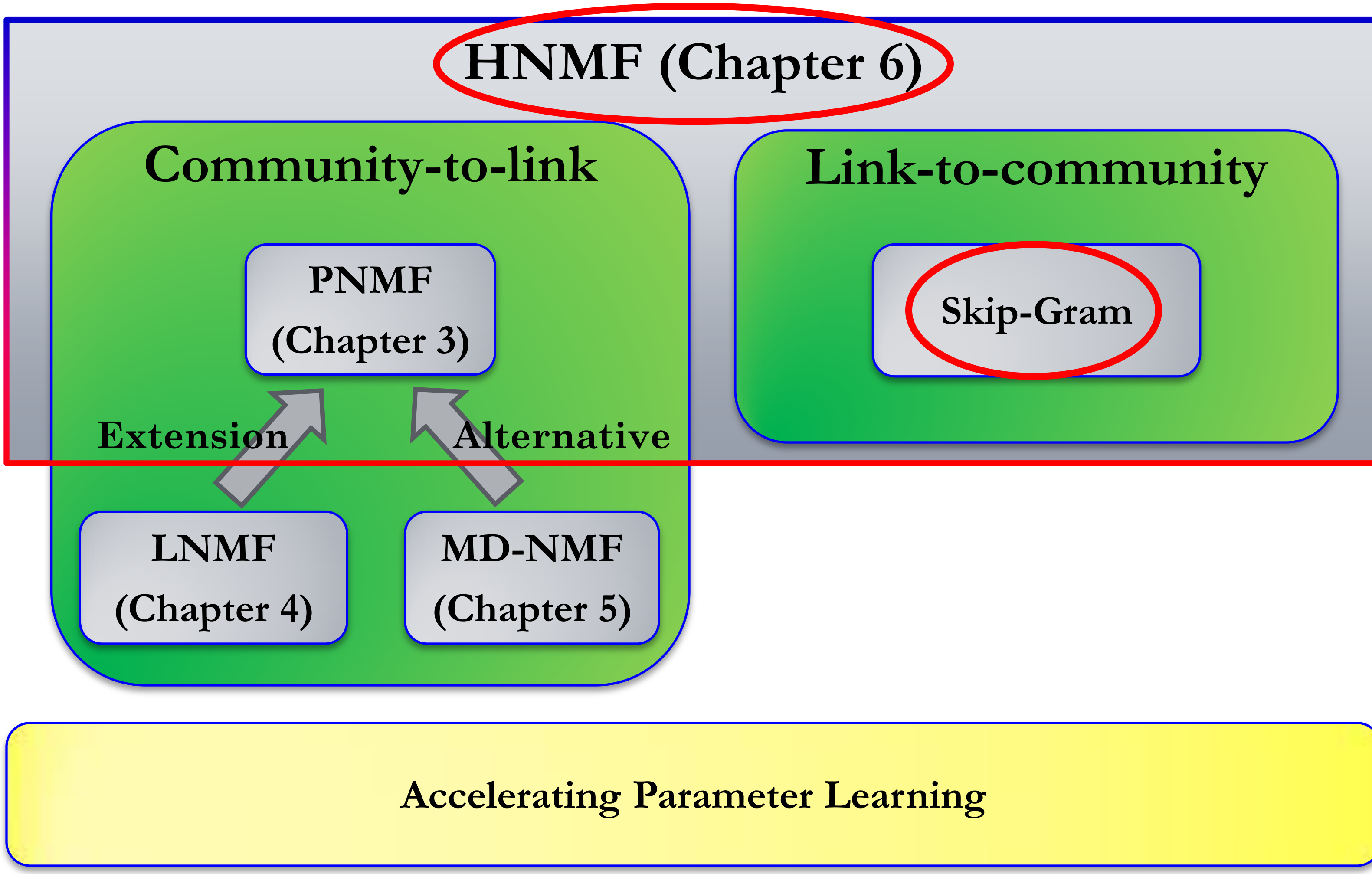
Table 3: **Comparison of experiment results in terms of modularity metric score.**

Dataset	SCP	BigCLAM	PNMF	MD-NMF
Amazon	0.0315	0.0441	0.0419	**0.0961**
DBLP	0.0967	0.0390	0.0985	**0.1013**

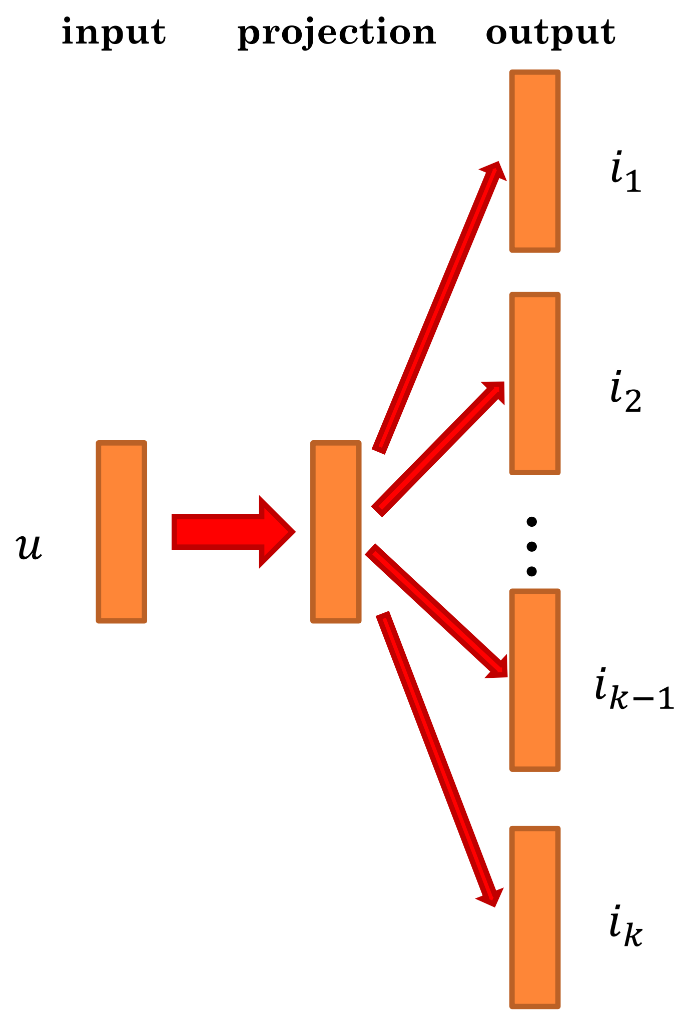
Table 4: **Comparison of experiment results in terms of F_1 score.**

- Baseline
  - Demon – Democratic estimate of the modular organization of a network [Coscia et al. 2012]

Research Work - Hierarchy



HNMF – Skip Gram



- [Mikolov et al. 2013]
- Assumption: the community membership of a node are similar to the community memberships of its neighbors.
- The learning objective is to maximize the sum of log-likelihoods for each node to represent its neighbors:

$$\sum_{u \in V} \sum_{i \in N^{+}(u)} \log \mathcal{P}(i|u).$$

HNMF – Skip Gram

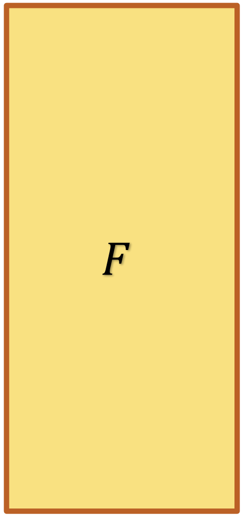
- The soft-max function is used to define $\mathcal{P}(i|u)$:

$$\mathcal{P}(i|u) = \frac{\exp(F_i F_u^T)}{\sum_{i'=1}^{|V|} \exp(F_{i'} F_u^T)}.$$

- Negative sampling is used to approximate the full soft-max function with less computational cost:

$$\mathcal{P}(i|u) = \sigma(F_i F_u^T) + h\mathbb{E}_{j\sim P_{N^-(u)}}\left[\sigma\left(-F_j F_u^T\right)\right],$$

where σ is the sigmoid function, h is the number of negative samples, and $P_{N^-(u)}$ is the uniform distribution from $N^-(u)$.



HNMF - FORMULATION

- A unified learning objective:

$$\boldsymbol{U}(\boldsymbol{F}) = \boldsymbol{C}(\boldsymbol{F}) + \boldsymbol{\beta}\boldsymbol{L}(\boldsymbol{F}) - \boldsymbol{\lambda}\boldsymbol{R}(\boldsymbol{F})$$

community-to-link

link-to-community

regularization term

relative importance coefficient

regularization coefficient

$$\boldsymbol{C}(\boldsymbol{F}) \coloneqq \sum_{u \in V} \log \mathcal{P}(>_u) = \sum_{u \in V} \sum_{i \in N^+(u), j \in N^-(u)} \log \mathcal{P}(r_{u,i} > r_{u,j})$$

$$\boldsymbol{L}(\boldsymbol{F}) \coloneqq \sum_{u \in V} \sum_{i \in N^+(u)} \log \mathcal{P}(i|u)$$

HNMF - Results

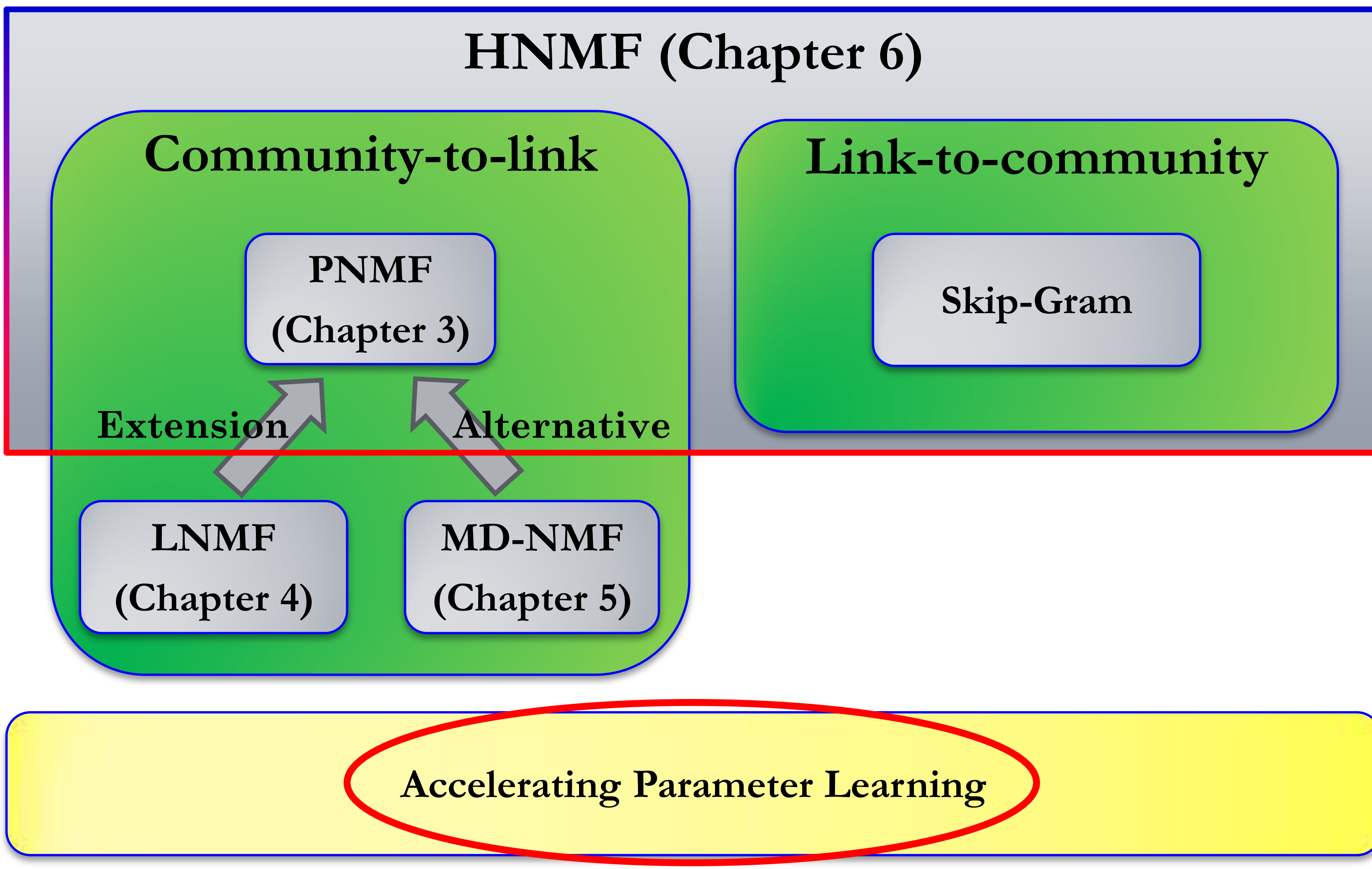
Dataset	SCP	Demon	BNMF	BNMTF	BigCLAM	PNMF	HNMF
Dolphins	0.305	0.680	0.507	0.507	0.423	0.979	**1.021**
Books about US politics	0.496	0.432	0.461	0.492	0.529	0.864	**0.988**
Word adjacencies	0.071	0.032	0.254	0.268	0.231	0.668	**0.699**
American college football	0.605	0.540	0.558	0.573	0.518	1.049	**1.113**
Power grid	0.044	0.195	0.342	0.368	1.010	1.105	**1.135**
High-energy theory	0.543	0.962	0.565	0.600	0.964	0.973	**1.060**

Table 5: Experimental results on Newman's networks in terms of modularity.

Dataset	Demon	BigCLAM	PNMF	HNMF
Amazon	0.082	0.044	0.042	**0.122**
DBLP	0.102	0.039	0.098	**0.104**

Table 6: Experimental results on two large networks in terms of F_1 score.

RESEARCH WORK - HIERARCHY



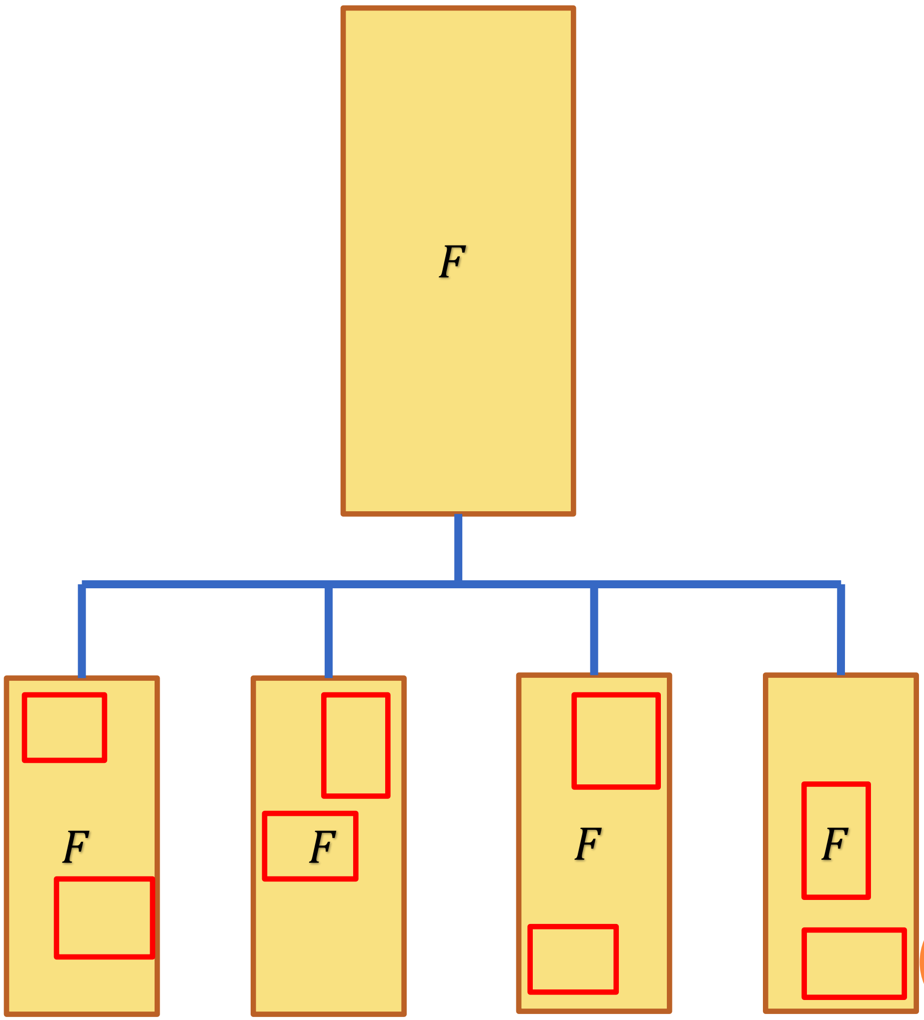
PARAMETERS LEARNING

- Projected stochastic gradient descent [Lin 2007] is used for parameter learning.
  - $\Theta \leftarrow \max(0, \Theta - \alpha \frac{\partial l}{\partial \Theta})$, Θ is any parameter and α is learning rate

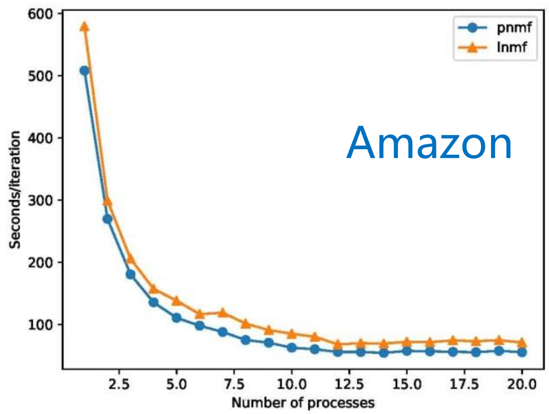
- In each iteration, we need to sample a set of tuples, which depends on the particular model.
  - PNMF: (source, neighbor, non-neighbor)
  - LNMF: (source, neighbor, local non-neighbor, distant non-neighbor)
  - MD-NMF: (source, α-acquaintance, β-stranger)
  - HNMF: (source, neighbor, non-neighbor, k negative samples from non-neighbors)

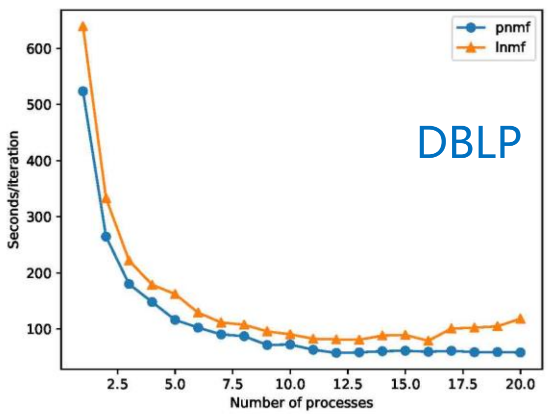
ACCELERATING PARAMETERS LEARNING

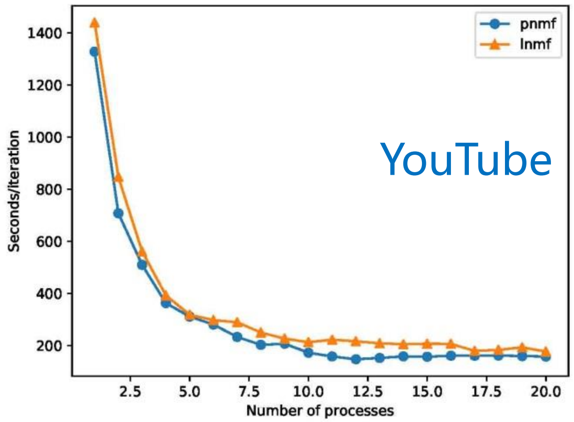
- Challenge:
  - Too many tuples, still spend too much time even with SGD
- Solution:
  - Multi-process with each threading dealing with a set of randomly generated samples
  - Lock-free parameter update



ACCELERATING PARAMETERS LEARNING







- Comparison between single-process implementation and multi-process implementation on three large datasets

OUTLINE

- Introduction and Motivation
- Background Study
  - Community Detection
  - Overlapping Community Detection (OCD)
  - Matrix Factorization (MF) framework for OCD
- Research Work
  - PNMF: Preference-based Non-negative MF Model
  - LNMF: Locality-based Non-negative MF Model
  - MD-NMF: Mutual Density-based Non-negative MF Model
  - HNMF: Homophily-based Non-negative MF Model
- Conclusion

CONCLUSION

- The first to explore and model both sides of mutual enhancement between links and communities for overlapping community detection.
- Four novel objective functions in non-negative matrix factorization framework for overlapping community detection.
- Better quality of detected communities (higher modularity and F_1 score), scalable to large datasets.

PUBLICATIONS/MANUSCRIPTS

- Conference
  - Xingyu Niu, **Hongyi Zhang**, Irwin King, Michael R. Lyu. From Mutual Friend to Overlapping Community Detection: a Non-negative Matrix Factorization Approach. To appear in 13th International Conference on Advanced Data Mining and Applications (ADMA' 17).
  - **Hongyi Zhang**, Tong Zhao, Irwin King, Michael R. Lyu. Modeling the Homophily Effect between Links and Communities for Overlapping Community Detection. In Proceedings of 25th International Joint Conference on Artificial Intelligence (IJCAI' 16), 3938-3944.
  - **Hongyi Zhang**, Michael R. Lyu, Irwin King. Exploiting k-Degree Locality to Improve Overlapping Community Detection. In Proceedings of 24th International Joint Conference on Artificial Intelligence (IJCAI' 15), 2394-2400.
  - **Hongyi Zhang**, Irwin King, Michael R. Lyu. Incorporating Implicit Link Preferences Into Overlapping Community Detection. In Proceedings of 29th AAAI Conference on Artificial Intelligence (AAAI' 15), 396-402.
- Journal
  - **Hongyi Zhang**, Xingyu Niu, Irwin King, Michael R. Lyu. Overlapping Community Detection with Preference and Locality Information: A Nonnegative Matrix Factorization Approach. Submitted to Journal of Data Mining and Knowledge Discovery (DATAMINE).

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