Problem 1

Design CFG and PDA for $L = \{0^n 1^n : n \geq 0\}$, where $\Sigma = \{0, 1\}$.

Solution

$$S \to 0S1 \mid \epsilon$$

and the PDA is as follows,
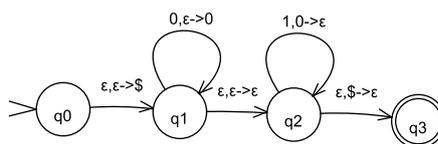
Problem 2

Show that $L = \{0^n 1^n 0^n : n \geq 0\}$ is not context-free, where $\Sigma = \{0, 1\}$.

Solution

It is not context-free. Suppose it is, by pumping lemma, there is a number p, choose $z = 0^p 1^p 0^p \in L$, z can be divided into five parts $z = uvxyz$ satisfying the following conditions: $|vy| > 0$, $|vxy| \leq p$ and for any $i \geq 0$, $uv^i xy^i z \in L_1$. Denote $z = z_1 z_2 z_3$, where $z_1 = z_3 = 0^p$ and $z_2 = 1^p$. Since $|vxy| \leq p$, we only need to consider the following two cases:

Case 1. vxy is contained in z_i for some $i = 1, 2, 3$, for instance $i = 1$, since $|vy| > 0$, then $uv^2 xy^2 z$ is still in $0^*1^*0^*$ but the number of 0's is more than $2p$ but the number of 1's is still p. It follows that $uv^2 xy^2 z \notin L$.

Case 2. vxy is contained in $z_i z_{i+1}$ for some $i = 1, 2$, for instance $i = 1$, since $|vy| > 0$, then uxz contains strictly less than $3p$, but the number of 0's in the end of the string $\geq p$. Then $uxz \notin L$.

Problem 3

Design CFG and PDA for $L = \{a^i b^j c^k d^l : i + k = j + l, i, k, j, l \geq 0\}$, where $\Sigma = \{a, b, c, d\}$.

Solution

$$S \to S_1 S_2 S_3 \mid aSd$$
$$S_1 \to aS_1 b \mid \epsilon$$
$$S_2 \to bS_2 c \mid \epsilon$$
$$S_3 \to cS_3 d \mid \epsilon$$

Here is the PDA.