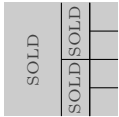

Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Is the following deduction rule valid?

$$\frac{\forall x: P(x) \text{ OR } Q(x)}{(\forall x: P(x)) \text{ OR } (\forall x: Q(x))}$$

2. Show that for every integer n , if $n^3 + n$ is divisible by 3 then $2n^3 + 1$ is *not* divisible by 3.
3. The vertices of graph G are the integers from 1 to 20. The edges of G are the pairs $\{x, y\}$ such that $\gcd(x, y) > 1$. How many connected components does G have?
4. What is $1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + 3 + \cdots + 1000)$?
5. An $n \times n$ plot of land (n is a power of two) is split in two equal parts by a North-South fence. The Western half is sold and the Eastern half is split in two equal parts by an West-East fence. The same procedure is applied to the remaining $(n/2) \times (n/2)$ plots until 1×1 plots are obtained (see $n = 4$ example). How many units of fence are used?
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6. A department has 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?
7. A password is made of the digits $0, 1, \dots, 9$ and the special symbols $*$ and $\#$. The password must be 4-6 symbols long and contain at least one special symbol. How many passwords are there?
8. Show that every set of 10 integers, each of them between 0 and 25, contains two distinct subsets S, T of the same size such that the sum of the numbers in S equals the sum of the numbers in T .