

1. The joint probability density function of the lifetimes  $X$  and  $Y$  of two connected components in a machine is

$$f_{X,Y}(x,y) = \begin{cases} xe^{-x(1+y)}, & x \geq 0, y \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) What is the probability that the lifetime  $X$  of the first component exceeds 3?
- (b) Are  $X$  and  $Y$  independent? Justify your answer.
2. A radio station gives a gift to the third caller who knows the birthday of the radio talk show host. Each caller has a 0.7 probability of guessing the host's birthday, independently of other callers.
- (a) What is the probability mass function of the number of calls necessary to find the winner?
- (b) What is the probability that the station will need five or more calls to find a winner?
3. Alice sends a message  $a$  that equals  $-1$  or  $1$ . Bob receives the value  $B$  which is a Normal random variable with mean  $a$  and standard deviation  $0.5$ . Bob guesses that Alice sent  $1$  if  $B > 0.5$ , that Alice sent  $-1$  if  $B < -0.5$ , and declares failure otherwise (when  $|B| \leq 0.5$ ).
- (a) What is the probability that Bob declares failure?
- (b) Given that Bob didn't declare failure, what is the probability that his guess is correct?
4. The number of people who enter an elevator on the ground floor is a Poisson random variable with mean  $10$ . There are  $20$  floors above (not including) the ground floor and each person is equally likely to get off on any one of them, independently of all others.
- (a) What is the probability  $p$  that the elevator doesn't stop on the seventh floor?
- (b) What is the expected number of stops that the elevator will make?  
(Express the answer in terms of  $p$  in case you didn't complete part (a).)
5. 500 balls are drawn without replacement from a bin with 600 black balls and 400 white balls.
- (a) What is the expected number of black balls drawn?
- (b) What is the variance of the number of black balls drawn?
- (c) Is the probability you drew fewer than 200 black balls more than 2%? Justify your answer.

$x :$	0	0.5	1	1.5	2	2.5	3	3.5
$P(X \leq x) :$	0.5	0.6915	0.8413	0.9331	0.9772	0.9938	0.9987	0.9998

CDF of a Normal(0, 1) random variable  $X$