

### Practice questions

1. Alice is mailing letters to solicit donations from CUHK alums. From past experience she knows that 30% of the alums make a 500 dollar donation, and 10% of the alums make a 1,000 dollar donation. Use the central limit theorem to estimate the number of letters Alice should mail to meet a 50,000 dollar donation target with probability 90%.
2. After much dating experience, Romeo concludes that girls show up  $\text{Exponential}(\Theta)$  hours late to a date, where  $\Theta$  is a  $\text{Uniform}(0, 1)$  random variable whose value is encoded deep in a girl's heart. On the first two dates Juliet shows up 10 minutes late and 30 minutes late. What is Romeo's posterior PDF for Juliet's  $\Theta$ ?
3. The ratio of A, B, and C students at CUHK is 1:2:2. The probability that A, B, and C students answer a true/false exam question correctly are 90%, 70%, and 55%, respectively. Bob takes a 4 question exam and answers 3 of them correctly. The teacher determines Bob's grade using a MAP estimate, assuming his answers are independent. What should Bob's grade be?
4. The lifetimes of a normal and a defective light bulb are exponential random variables with averages of 5 years and 2 years, respectively. On average, one of three light bulbs comes out defective.
  - (a) For which values of  $y$  is a light bulb that burns out in  $y$  years more likely to be defective than not under a MAP estimate?
  - (b) What is the estimation error (the probability of an incorrect conclusion) in part (a)?
5. In this question you will investigate sampling using Bayesian statistics. You have a coin of an unknown probability of heads  $P$ . Your prior is that  $P$  is a  $\text{Uniform}(0, 1)$  random variable.
  - (a) The coin is flipped 10 times and 9 of the 10 flips are heads. What is the posterior probability that  $P > 80\%$ ?
  - (b) (**Optional**) Let  $P$  indicate the probability that a triangle whose vertex coordinates are independent  $\text{Uniform}(0, 1)$  random variables is acute. Estimate  $P$  by running a computer simulation 500 times. What is the probability that  $P$  is within 5% of your estimate?
  - (c) A second coin, whose prior is also uniform and independent of the first coin, is flipped 10 times and all flips are heads. What is the probability that the second coin is more biased towards heads than the first one?