

### Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. Let  $X$  be an Exponential( $\lambda$ ) random variable. Find the PDF of the random variables (a)  $Y = X^2$  and (b)  $Z = e^{-\lambda X}$ .
2. Raindrops hit the ground at a rate of 1 per second. An observatory has a raindrop sensing equipment. A signal is received by the computer with a maximum delay of 1 second after sensing a raindrop, with all delays equally likely. Find
  - (a) The joint PDF of the time  $T$  of the first raindrop and the time  $S$  of the signal reception.
  - (b) The marginal PDF of  $S$ .
  - (c) The conditional PDF of  $T$  given  $S$ .
3. The body temperatures of a healthy person and an infected person are Normal(36.8, 0.5) and Normal(37.8, 1.0) random variables, respectively. About 1% of the population is infected. What is the conditional probability that I am infected given that my temperature is  $t$ ? For which values of  $t$  am I more likely to be infected than not?
4. Raindrops hit your head at a rate of 1 per second. What is the PDF of the time at which the second raindrop hits you? How about the third one? (**Hint:** convolution)
5. You draw 10 balls at random among 15 red and 5 blue balls without replacement. Let  $X$  be the number of red balls drawn.
  - (a) What is the expected value of  $X$ ?
  - (b) Write  $X = X_1 + X_2 + \cdots + X_{10}$ , where  $X_i$  indicates if the  $i$ -th drawn ball is red. What is the variance of  $X_i$ ?
  - (c) What is the covariance of  $X_i$  and  $X_j$  ( $i \neq j$ )?
  - (d) What is the variance of  $X$ ?