

Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. A point is chosen uniformly at random inside a triangle with base 1 and height 1. Let X be the distance from the point to the base of the triangle. Find the CDF and the PDF of X . (*Textbook problem 3.2.5*)
2. There are 100 students in class. The arrival times of students (in minutes) are exponential random variables with rate $\lambda = 0.2$, starting from 09:20.
 - (a) What is the expected number of students that have arrived by 09:30?
 - (b) Assuming students' arrivals are independent, what is the probability that everyone has made it by 09:45?
3. Three points are dropped at random on the perimeter of a circle with 1 unit circumference.
 - (a) What is the probability that they all fall within $1/4$ of a unit of one another?
 - (b) What is the probability that every pair of them is at least $1/4$ of a unit apart? (**Hint:** Fix one of the three points.)
4. A coin has probability P of being heads, where P itself is a Uniform(0, 1) random variable. Find the PMF of the number of heads after performing two independent coin flips.
5. Here is a way to solve Buffon's needle problem without calculus. Recall that an ℓ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.
 - (a) Let A be the number of lines that the needle hits. Let B be the number of times that a polygon of perimeter ℓ hits a line. Show that $E[A] = E[B]$. (**Hint:** Use linearity of expectation.)
 - (b) Assume that $\ell < \pi$. Calculate the expected number of times that a circle of perimeter ℓ hits a line.
 - (c) Assume that $\ell < 1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. (**Hint:** The number of hits is a 0/1-valued random variable.)