

### Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. Roll a 3-sided die twice. Let  $X$  be the larger number and  $Y$  be the smaller number you rolled. Find (a) the conditional PMF of  $X$  given  $Y$  and (b) the expected value of  $X$  given  $Y = y$  for all values of  $y$ .
2. In 2017 there were 0.848 men for every woman in Hong Kong. Men and women had life expectancies of 81.7 years and 87.7 years, respectively. What was the life expectancy of a random person?
3. You toss a coin 100 times. Which of the following random variables are independent?
  - (a) The number of consecutive heads  $HH$  and the number of consecutive tails  $TT$ .
  - (b) The number of consecutive heads in the first 50 tosses and the number of consecutive tails in the last 50 tosses.
  - (c) The random variables in part (b), conditioned on having exactly 50 heads in the 100 coin tosses.
4. You go to the casino with \$3 to play roulette. (Roulette has 37 possible outcomes, out of which 18 are red, 18 are black, and one is green.) Calculate the expected value and standard deviation of your profit under the following two gambling strategies:
  - (a) You play for 3 rounds, where in every round you bet \$1 on red.
  - (b) You bet all your money on red. If you win, you bet everything on red again. If you win again, you bet everything on red one last time.
5. Consider 10 persons forming 5 couples who live together at a given time. Suppose that at some later time, the probability of each person being alive is  $p$ , independent of other persons. At that later time, let  $A$  be the number of persons that are alive and let  $S$  be the number of couples in which both partners are alive. Find  $E[S \mid A = a]$ . (*Textbook problem 2.32*)