

Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. Roll a 3-sided die twice. Let X be the larger number and Y be the smaller number you rolled. Find the joint PMF of X and Y , their marginal PMFs, and the expected value of $X + Y$.
2. On any given day between Monday and Saturday, the probability that you'll have a late snack is 20%, independent on the other days. You'll have a late snack on Sunday *if and only if* you didn't have one in any of the previous six days. What is the expected number of snacks you'll be having?
3. Let p be a number between 0 and 1. Toss a p -biased coin. If the coin comes up heads, toss a fair coin and report the outcome twice (1 for heads, 0 for tails). If the coin comes up tails, report the outcomes of two independent fair coin tosses. Show that the marginal PMFs of your two reports are the same for every p , but the joint PMFs are all different.
4. Alice has two identical envelopes containing \$1 and \$2 respectively. She shows a random envelope to Bob, who guesses the amount of money in the envelope and then opens it. If Bob's guess is correct he collects the money in the envelope. If not, Bob gets nothing. Suppose Alice picks the \$1 envelope with probability p and the \$2 envelope with probability $1 - p$. Assuming Bob knows the value of p , how should Bob guess (as a function of p)?
5. 100 balls are tossed at random into 100 bin. Each ball is equally likely to land in any of the bins, independently of the other balls. What is the expected number of bins that receive exactly one ball?