

### Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. A power utility can supply electricity to a city from  $n$  different power plants. Power plant  $i$  fails with probability  $p_i$ , independent of the others.
  - (a) Suppose that any one plant can produce enough electricity to supply the entire city. What is the probability that the city will experience a black-out?
  - (b) Suppose that all  $n$  plants are necessary to produce enough electricity to supply the entire city. What is the probability that the city will experience a black-out?

*(Adapted from Textbook problem 1.36)*

2. An ENGG 2430A tutorial meets for 11 weeks. Each week, the TA asks 5 questions and chooses 5 random but distinct students to answer them, independently of what happened in previous weeks. If you are one of 30 students in the tutorial (and attendance is always perfect!), what is the probability that you are chosen in the final week but not before that?
3. You go to a party with 500 guests.
  - (a) What is the probability that exactly one other guest has the same birthday as you? (For simplicity, exclude birthdays on February 29.)
  - (b) Now model the number of other guests that share your birthday as a Poisson( $\lambda$ ) random variable  $N$ . What is the rate  $\lambda$ ? How does the probability that  $N$  equals 1 compare to the answer in part (a)?

*(Adapted from Textbook problem 2.2)*

4. Alice, Bob, and Charlie are equally likely to have been born on any three days of the year. Let  $E_{AB}$  be the event that Alice and Bob were born on the same day. Define  $E_{BC}$  and  $E_{CA}$  analogously. Which of the following statements is true:
  - (a) Any two of the three events  $E_{AB}, E_{BC}, E_{CA}$  are independent.
  - (b)  $E_{AB}, E_{BC}$ , and  $E_{CA}$  are independent.
  - (c)  $E_{AB}$  and  $E_{BC}$  are independent conditioned on  $E_{CA}$ .
5. You flip two fair coins. If they both come out heads you stop. If not, you try again until they do. Let  $F$  be the total number of coin flips you performed. For example if the outcome is THHTHH then  $F = 6$ . What is the PMF (probability mass function) of  $F$ ?