## Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. In how many ways can we roll 4 dice so that
   (a) The face values of the dice are all different?
   (b) The face values of the dice are increasing (e.g., 2356 but not 3516, 1224)?
2. A bin contains 10 black balls and 10 white balls. You draw three balls without replacement. What is the probability that all three are black?
3. ENGG 2760A has 100 students this year, including Alice and Bob. The students are randomly divided into three tutorials with 30, 35, and 35 students, respectively.
   (a) What is the probability that Alice and Bob are both assigned to the 30-student tutorial?
   (b) What is the probability that Alice and Bob are assigned to the same tutorial?
4. A six-sided die is rolled three times. Which is more likely: A sum of 11 or a sum of 12? *(Textbook problem 1.49)*

## Additional ESTR 2018 questions

Feel free to use any means at your disposal (mathematical analysis, computer experiments, material from external sources) to tackle these questions. Sample answers will not be provided. The additional questions can serve as a starting point for the ESTR 2018 final projects.

5. In this question you will study variants of the "birthday paradox" from lecture.
   (a) Can you derive a formula for the probability that among $n$ (random) students some three of them were born on the same day?
   (b) Assume a year on planet X has $m$ days. Can you write a program that calculates the probability $p(n, m, k)$ that among $n$ students on planet X, some $k$ of them were born on the same day?
   (c) Let $f(m, k)$ be the smallest $n$ for which $p(n, m, k)$ exceeds 1/2. Can you come up with a simple formula that approximates $f(m, 2)$ when $m$ is large? How about $f(m, 3)$?
6. A particle sits in one of seven equally spaced slots along a circle. At each step the particle is equally likely to move into one of the two adjacent slots. Let $p(t)$ be the probability that the particle is back where it started after $t$ steps.
   (a) Derive a formula for $p(t)$.
   (b) Which value $p_\infty$ does $p(t)$ should converge to when $t$ goes to infinity?
   (c) Is $p(t)$ increasing with $t$, decreasing with $t$, or neither?
   (d) What is the smallest $t$ for which $0.1 \leq p(t) \leq 0.2$? More generally, what can you say about the smallest $t$ for which $p_\infty - \varepsilon \leq p(t) \leq p_\infty + \varepsilon$?