Jason is taking `ENGG2430` and `CSCI2100` this semester. For each course, he estimates his probability of getting an A to be 50%. He also thinks there is a 30% probability that he ends up with exactly one A. What is the probability that he gets As in both courses?

Solution: Let E and F be the events that Jason gets an A in `ENGG2430` and `CSCI2100`, respectively. We are given that $\mathrm{P}(E) = \mathrm{P}(F) = 50\%$ and $\mathrm{P}(E \setminus F) + \mathrm{P}(F \setminus E) = 30\%$, and we want to know the probability of $E \cap F$. By the axioms of probability,

$$\mathrm{P}(E) = \mathrm{P}(E \setminus F) + \mathrm{P}(E \cap F) \qquad \text{and} \qquad \mathrm{P}(F) = \mathrm{P}(F \setminus E) + \mathrm{P}(E \cap F).$$

Therefore

$$2\,\mathrm{P}(E \cap F) = \mathrm{P}(E) + \mathrm{P}(F) - (\mathrm{P}(E \setminus F) + \mathrm{P}(F \setminus E)) = 2 \cdot 50\% - 30\% = 70\%,$$

so $\mathrm{P}(E \cap F) = 35\%$.