Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. Suppose $\mathrm{P}(A) = 3/4$ and $\mathrm{P}(B) = 1/3$. Show that $1/12 \leq \mathrm{P}(A \cap B) \leq 1/3$.
2. A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. What is the probability that the outcome is less than 4? *(Textbook problem 1.6)*
3. A bin contains 50 black balls and 50 white balls. You draw two balls without replacement. What is the probability that at least one of them is white?
4. A six-sided die is rolled three times. Which is more likely: A sum of 11 or a sum of 12? *(Textbook problem 1.49)*
5. A bin contains 3 white balls and 5 black balls. Alice and Bob take turns drawing balls from the bin without replacement until a white ball is drawn. Assuming Alice goes first, what is the probability that she gets the white ball?

Additional ESTR 2002 questions

6. The frequentist interpretation of probability says that the probability of an event is the fraction of times it occurs if the experiment is repeated many times. Test this hypothesis by writing and running a computer program for the event in question 5 above. How many runs of the experiment do you roughly need so that the probability you calculated in question 5 and the fraction of times your program succeeds match in the first three digits?
7. You draw n balls without replacement from a bin with n black balls and n white balls. Let $p(n)$ be the probability that you drew an even number of white balls. Investigate the asymptotic order of growth of the function $|p(n) - 1/2|$ as well as you can (e.g., is it $O(1/n)$? $2^{-O(n)}$? $O(e^{-n})$?). Empirical, theoretical, and simulation-based analyses are acceptable.