ENGG 2760A / ESTR 2018: Probability for Engineers

Tutorial 10

1. Total probability theorem vs Convolution
2. Quiz 7&8
3. Homework 11
4. Q&A (HW10)

Total Probability Theorem VS. Convolution

Total probability theorem: → Calculate the marginal PMF/PDF of multiple random variables.

$f_X(x) = \sum_y f_{X|Y}(x \mid y)\, f_Y(y)$ Y **discrete**

$f_X(x) = \int f_{X|Y}(x \mid y)\, f_Y(y)\, dy$ Y **continuous**

Convolution → Calculate the PMF/PDF of the sums of independent random variables.

X, Y are independent.
What is the PMF/PDF of $X + Y$? Z

PMF: $P(Z=z) = \sum_{x,y:\,x+y=z} P(X=x, Y=y)$

$= \sum_{x,y:\,x+y=z} P(X=x)P(Y=y)$

$= \sum_x P(X=x)P(Y=z-x)$

$$\boxed{\begin{aligned} f_Z(z) &= \sum_x f_X(x) f_Y(z-x) && \text{DISCRETE} \\ f_Z(z) &= \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx && \text{CONTINUOUS} \end{aligned}}$$

Cite:
20L08(link:http://www.cse.cuhk.edu.hk/~andrejb/engg2760/slides/20L08.pptx)
20L09(link:http://www.cse.cuhk.edu.hk/~andrejb/engg2760/slides/20L09.pptx)

Quiz 7.

Alice is about to take two laps in the CUHK swimming pool. The time of her first lap is F minutes, where F is an Exponential(1) random variable. The time of her second lap is S minutes, where S is an Exponential(F) random variable. What is the probability that she completes her second lap within one minute?

Q1 : How to understand Exponential (F) ?

$$\text{Exponential}(F) = Fe^{-Fs} \quad s \geq 0$$

→ the conditional Pro. of S given F

$P(F)$ $E(F)$ X

F : the time of first lap

$$P(S=s \mid F=f) = \begin{cases} fe^{-fs} & s \geq 0 \\ 0 & o/w \end{cases}$$

② Given that the first time is F

Q2: final solving pro.

$$P(S \leq 1) \longrightarrow P(T \leq F+1 \mid T \geq F) \Leftarrow \int_F P(T \leq F+1 \mid T \geq F)\, dF$$

Total time : $T = S + F$

①

S, F are not independent ~~convolution~~

$P(T)$

Quiz 8:

Five boy-girl couples arrive in a restaurant. The ten guests take up random places at a round table. Let N be the number of couples that end up in adjacent seats. What is the variance of N?

$$N = N_1 + N_2 + \cdots + N_5$$

$$N_i = \begin{cases} 1 & i\text{-th couple sit next to each other (Prob. } \frac{2}{9}) \\ 0 & \text{o/w} \end{cases}$$

$$\mathrm{Var}[N] = \sum_i \mathrm{Var}[N_i] + \sum_{i \neq j} \mathrm{Cov}[N_i, N_j]$$

$$= 5 \times \left(\frac{2}{9} - \left(\frac{2}{9}\right)^2 \right) + 5 \times 4 \times \left(\underbrace{\frac{2}{9}\left(\frac{2}{8} \times \frac{1}{7} + \frac{6}{8} \times \frac{2}{7}\right)}_{E[N_iN_j]} - \underbrace{\left(\frac{2}{9}\right)^2}_{E[N_i]E[N_j]} \right)$$

$$= \frac{80}{81}$$