Week 10 Tutorial Session

(1) In this problem, you will design Turing machine for the following language:

$$L = \{\mathtt{a}^n \# \mathtt{a}^n \# \mathtt{a}^n \colon n \geq 0\},\ \Sigma = \{\mathtt{a}, \#\}.$$

Give a high-level description of your Turing machine. If you have time left after finishing the next problem, give also a state diagram.

(2) A **Turing machine with doubly infinite tape** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward.
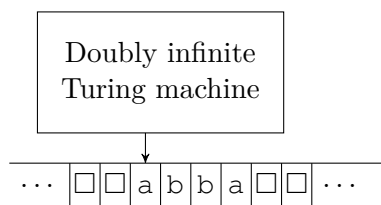
You will argue that this type of Turing machine is equivalent to the usual one-side-unbounded Turing machine.

(a) Write a formal definition of a doubly infinite tape Turing machine. A formal definition of an automaton will look like page 17/slide 16 of Lecture 14 or page 9/slide 7 of Lecture 10.

(b) Show how to simulate a usual Turing machine on a doubly infinite tape Turing machine. You need to specify

- how the tape of the doubly infinite Turing machine will be used to represent the usual Turing machine.
- how the doubly infinite Turing machine tape should be set up initially;
- what the doubly infinite Turing machine should do when the usual Turing machine performs a transition (you may specify in 1-2 sentences the general idea, omitting the tedious details);
- what the doubly infinite Turing machine should do when the usual Turing machine accepts/rejects.

(c) Show how to simulate a doubly infinite Turing machine on a usual Turing machine. Again you should specify simulation details similar to those in part (b). *Hint:* You may want to simulate the doubly infinite Turing machine with some machine M that is not the usual Turing machine, but can itself be simulated by a usual Turing machine.

(3) (We will not have time to go over this question during the tutorial session. This question is included here only because it is similar to a homework question.)

Argue that the collection of decidable languages is closed under intersection.