**Week 5 Tutorial Session**

1. For an integer $k \geq 1$, define $L_k$ to be the set of strings (over $\Sigma = \{\texttt{0}, \texttt{1}\}$) that have a `1` at the $k$th-to-last position. For example, **`1`**`00` and `0`**`1`**`01` are in $L_3$, but `0` and `011` are not.

   (a) Prove that every DFA for $L_k$ has at least $2^k$ states.

   (b) Describe (e.g. with a diagram) an NFA for $L_k$ that has at most $k + 1$ states.

2. Let $L$ be the set of strings over $\{\texttt{0}, \texttt{1}\}$ whose number of ones is a perfect square (e.g. $0, 1, 4, 9, 16, \ldots$). Prove that $L$ is irregular.