Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (48 points) Give a DFA for the following languages, specified by a transition diagram. For each one of them, briefly describe how the DFA works, by stating in English what strings stop at every state. The alphabet is $\Sigma = \{0, 1\}$.

    (a) $L_1 = \{w \mid w$ contains exactly two 1's$\}$

    (b) $L_2 = \{w \mid w$ does not contain the substring $010\}$

    (c) $L_3 = \{w \mid$ Every even position of w is a $0\}$
        The first symbol of a string is an odd position, so 10 should be accepted.

    (d) $L_4 = \{w \mid w$ contains an odd number of 0's, or contains exactly two 1's$\}$

(2) (12 points) Convert the following NFA to a DFA using the method described in class. Specify the DFA by its transition diagram. The alphabet is $\Sigma = \{0, 1\}$.
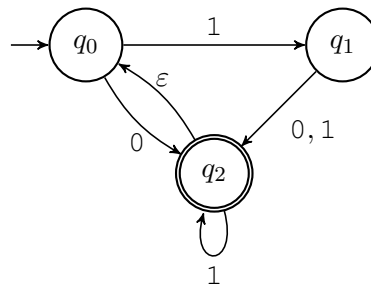
(3) (20 points) If w is a string, call a string x a *subsequence* of w if x can be obtained from w by removing zero or more symbols from w. For example, bd and c are both subsequences of bcde.

Formally, $x = x_1 \cdots x_m$ is a subsequence of w if $w = s_0 x_1 s_1 x_2 \cdots x_m s_m$ for some strings $s_0, \cdots, s_m$.

Given a language L, define $L^S = \{x \mid x$ is a subsequence of some $w \in L\}$. That is, L^S contains all the subsequences of strings in L.

Prove that if L is a regular language, then so is L^S.

Hint: Regular language is defined *recursively*. If the desired result is true for simpler regular languages, can you show that it is also true for more complex regular languages?

(4) (20 points) Consider the following DFA:
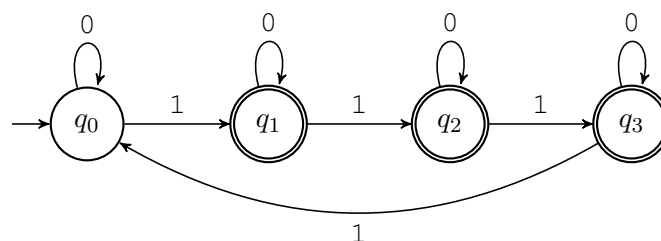
(a) What strings stop at q_0? At q_1? At q_2? At q_3?

(b) State an induction hypothesis that will allow you to prove your answer in (a).

(c) What is the language of the DFA?