LR(0) Parsers

CSCI 3130 Formal Languages and Automata Theory

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Fall 2015

Parsing computer programs

```
if (n == 0) { return x; }
```

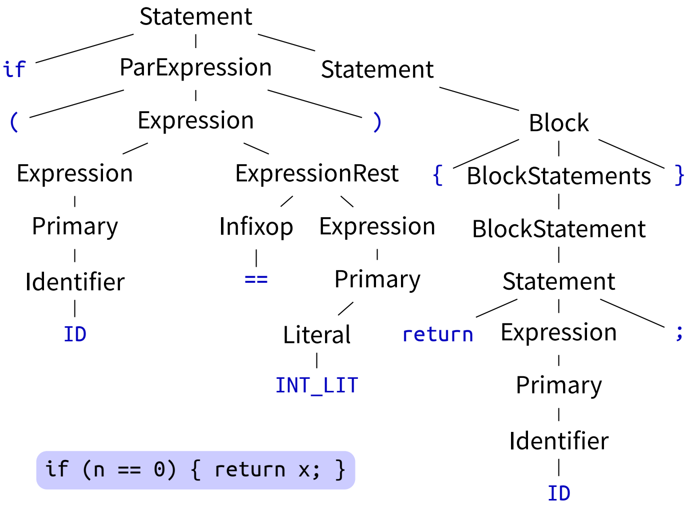
First phase of `javac` compiler: lexical analysis

$\boxed{\texttt{if}}$ $\boxed{\texttt{(}}$ $\boxed{\texttt{ID}}$ $\boxed{\texttt{==}}$ $\boxed{\texttt{INT\_LIT}}$ $\boxed{\texttt{)}}$ $\boxed{\texttt{\{}}$ $\boxed{\texttt{return}}$ $\boxed{\texttt{ID}}$ $\boxed{\texttt{;}}$ $\boxed{\texttt{\}}}$

The alphabet of Java CFG consists of tokens like

$$\Sigma = \{\texttt{if}, \texttt{return}, \texttt{(}, \texttt{)}, \texttt{\{}, \texttt{\}}, \texttt{;}, \texttt{==}, \texttt{ID}, \texttt{INT\_LIT}, \ldots\}$$

Parsing computer programs



Parse tree of a Java statement

CFG of the java programming language

```
Identifier:
    IdentifierChars but not a Keyword or BooleanLiteral or NullLiteral
Literal:
    IntegerLiteral
    FloatingPointLiteral
    BooleanLiteral
    CharacterLiteral
    StringLiteral
    NullLiteral
Expression:
    LambdaExpression
    AssignmentExpression
AssignmentOperator:
    (one of) = *= /= %= += -= <<= >>= >>>= &= ^= |=
```

from http://java.sun.com/docs/books/jls/second_edition/html/syntax.doc.html#52996

Parsing Java programs

```
class Point2d {
    /* The X and Y coordinates of the point--instance variables */
    private double x;
    private double y;
    private boolean debug;    // A trick to help with debugging

    public Point2d (double px, double py) { // Constructor
  x = px;
  y = py;

  debug = false;     // turn off debugging
    }

    public Point2d () {   // Default constructor
  this (0.0, 0.0);                    // Invokes 2 parameter Point2D constructor
    }
    // Note that a this() invocation must be the BEGINNING of
    // statement body of constructor

    public Point2d (Point2d pt) {             // Another consructor
  x = pt.getX();
  y = pt.getY();
    }
  ...
}
```

Simple Java program: about 1000 tokens

Parsing algorithms

How long would it take to parse this program?

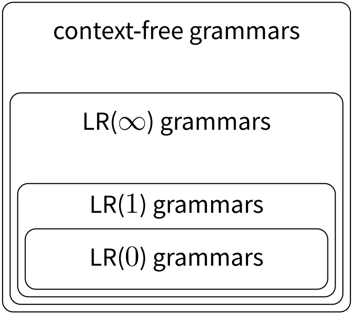
try all parse trees	$\geqslant 10^{80}$ years
CYK algorithm	hours

Can we parse faster?

CYK is the fastest known general-purpose parsing algorithm for CFGs

Luckily, some CFGs can be rewritten to allow for a faster parsing algorithm!

Hierarchy of context-free grammars



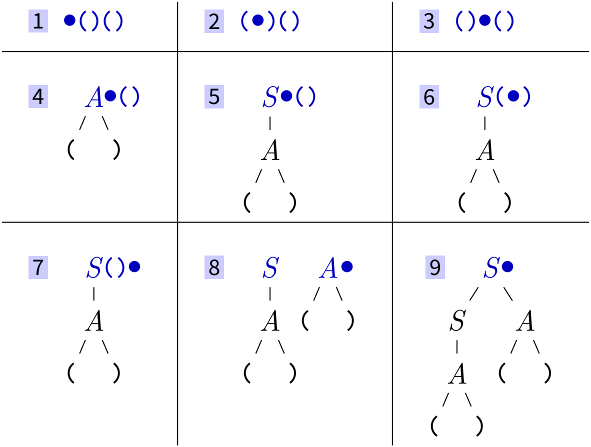
Java, Python, etc have LR(1) grammars

We will describe LR(0) parsing algorithm
A grammar is LR(0) if LR(0) parser works correctly for it

LR(0) parser: overview

$$S \to SA \mid A$$
$$A \to (S) \mid ()$$

input: ()()



LR(0) parser: overview

$$S \to SA \mid A$$
$$A \to (S) \mid ()$$

input: ()()

Features of LR(0) parser:

- Greedily reduce the recently completed rule into a variable
- Unique choice of reduction based on what has been read so far

```
3  ()•()   ⇒   4    A•()    ⇒   5    S•()
                   / \                |
                  (   )               A
                                     / \
                                    (   )
```

LR(0) parsing using a PDA

To speed up parsing, keep track of partially completed rules in a PDA P

In fact, the PDA will be a simple modification of an NFA N

The NFA accepts if a rule $B \to \beta$ has just been completed
and the PDA will reduce β to B

$\ldots \Rightarrow$ 2 (•)() $\Rightarrow$ 3 ()•() $\stackrel{\checkmark}{\Rightarrow}$ 4 A•() $\stackrel{\checkmark}{\Rightarrow}$ 5 S•() $\Rightarrow \ldots$

```
   A•()            S•()
  /   \            |
 (     )           A
                  /   \
                 (     )
```

$\checkmark$: NFA N accepts

NFA acceptance condition

$$S \to SA \mid A$$
$$A \to \texttt{(}S\texttt{)} \mid \texttt{()}$$

A rule $B \to \beta$ has just been completed if

Case 1 input/buffer so far is exactly β

Examples: 3 ()•() and 4 A•()

```
 A•()
 / \
(   )
```

Case 2 Or buffer so far is $\alpha\beta$ and there is another rule $C \to \alpha B \gamma$

Example: 7 S()•

```
 S()•
 |
 A
/ \
(   )
```

This case can be chained

Designing NFA for Case 1

$$S \rightarrow SA \mid A$$
$$A \rightarrow (S) \mid ()$$

Design an NFA N' to accept the right hand side of some rule $B \rightarrow \beta$

Designing NFA for Case 1

$$S \to SA \mid A$$
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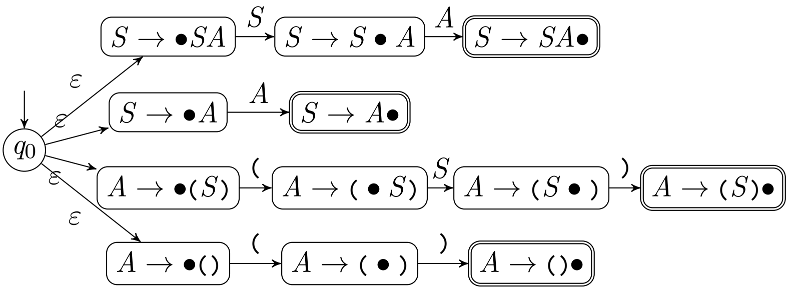
Design an NFA N' to accept the right hand side of some rule $B \to \beta$



Designing NFA for Cases 1 & 2

$$S \to SA \mid A$$
$$A \to \texttt{(}S\texttt{)} \mid \texttt{()}$$

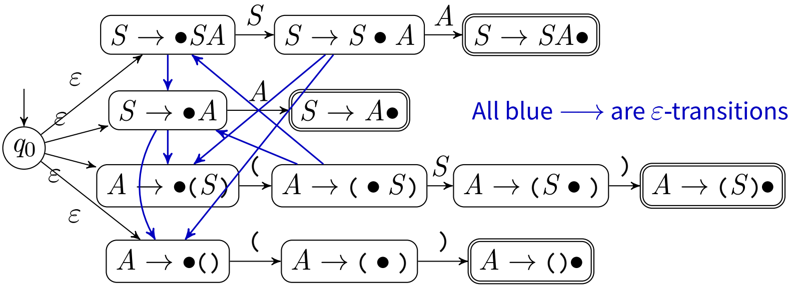
Design an NFA N to accept $\alpha\beta$ for some rules
$C \to \alpha B\gamma, \quad B \to \beta$
and for longer chains

Designing NFA for Cases 1 & 2

$$S \to SA \mid A$$
$$A \to (S) \mid ()$$

Design an NFA N to accept $\alpha\beta$ for some rules
$C \to \alpha B\gamma, \quad B \to \beta$
and for longer chains

For every rule $C \to \alpha B\gamma$, $B \to \beta$, add $\boxed{C \to \alpha \bullet B\gamma} \xrightarrow{\varepsilon} \boxed{B \to \bullet\beta}$



All blue $\longrightarrow$ are ε-transitions

Summary of the NFA

For every rule $B \to \beta$, add

$$\longrightarrow q_0 \xrightarrow{\varepsilon} \boxed{B \to \bullet\beta}$$

For every rule $B \to \alpha X \beta$ (X may be terminal or variable), add

$$\boxed{B \to \alpha \bullet X\beta} \xrightarrow{X} \boxed{B \to \alpha X \bullet \beta}$$

Every completed rule $B \to \beta$ is accepting

$$\boxed{\boxed{B \to \beta\bullet}}$$

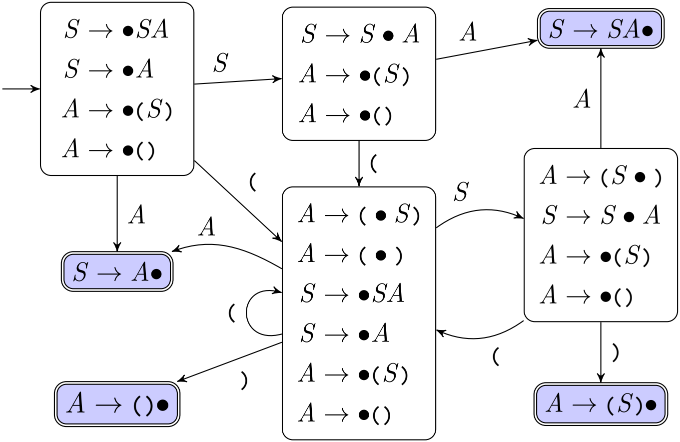
For every rule $C \to \alpha B \gamma$, $B \to \beta$, add

$$\boxed{C \to \alpha \bullet B\gamma} \xrightarrow{\varepsilon} \boxed{B \to \bullet\beta}$$

The NFA N will accept whenever a rule has just been completed

Equivalent DFA D for the NFA N

Dead state (empty set) not shown for clarity



Observation: every accepting state contains only one rule:
a completed rule $B \to \beta\bullet$, and such rules appear only in accepting states

LR(0) grammars

A grammar G is LR(0) if its corresponding D_G satisfies:

Every accepting state contains only one rule:
a completed rule of the form $B \to \beta\bullet$
and completed rules appear only in accepting states

Shift state:

no completed rule

$$\boxed{\begin{array}{l} S \to S \bullet A \\ A \to \bullet(S) \\ A \to \bullet() \end{array}}$$

Reduce state:

has (unique) completed rule

$$\boxed{A \to (S)\bullet}$$

Simulating DFA D

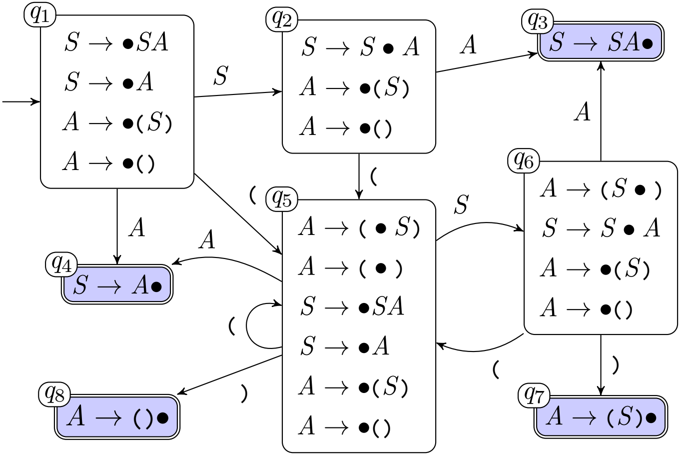
Our parser P simulates state transitions in DFA D

```
(()•)      ⇒      ( A •)
                   / \
                  (   )
```

After reducing `()` to A, what is the new state?

Solution: keep track of previous states in a stack
go back to the correct state by looking at the stack

Let's label D's states



LR(0) parser: a "PDA" P simulating DFA D

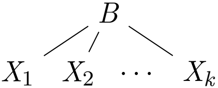
P's stack contains labels of D's states to remember progress of partially completed rules

At D's non-accepting state q_i

1. P simulates D's transition upon reading terminal or variable X
2. P pushes current state label q_i onto its stack

At D's accepting state with completed rule $B \to X_1 \dots X_k$

1. P pops k labels $q_k, \dots, q_1$ from its stack
2. constructs part of the parse tree



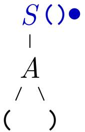
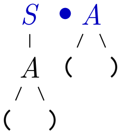
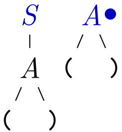
3. P goes to state q_1 (last label popped earlier), pretend next input symbol is B

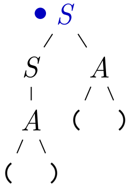
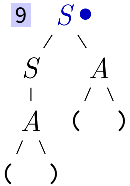
Example

		state	stack
1	•()()	q_1	\$
2	(•)()	q_5	\$1
3	()•()	q_8	\$15
	•A() / \ ( )	q_1	\$
4	A•() / \ ( )	q_4	\$1
	• S() | A / \ ( )	q_1	\$

		state	stack
5	S•() | A / \ ( )	q_2	\$1
6	S(•) | A / \ ( )	q_5	\$12

Example

		state	stack
7		q_8	$125
		q_2	$1
8		q_3	$12

		state	stack
		q_1	$
9		q_2	$1

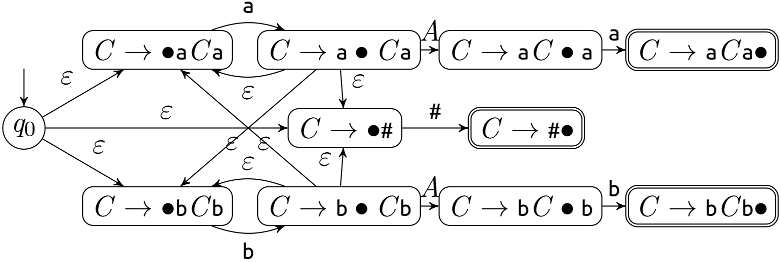
parser's output is the parse tree

Another LR(0) grammar

$$L = \{w\texttt{\#}w^R \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$$

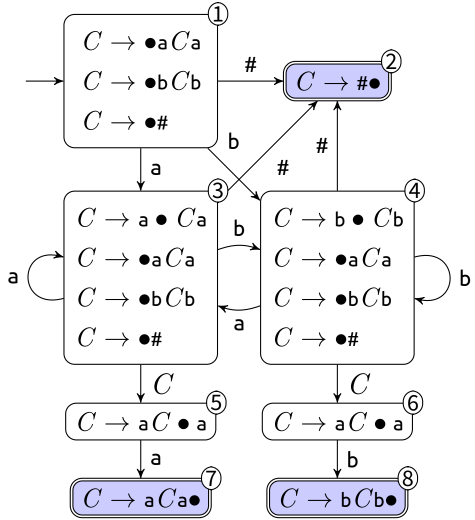
$$C \to \mathtt{a}C\mathtt{a} \mid \mathtt{b}C\mathtt{b} \mid \texttt{\#}$$

NFA N:



Another LR(0) grammar

$$C \rightarrow \mathtt{a}\,C\,\mathtt{a} \mid \mathtt{b}\,C\,\mathtt{b} \mid \#$$



input: ba#ab

stack	state	action
$	1	S
$1	4	S
$14	3	S
$143	2	R
$143	5	S
$1435	7	R
$14	6	S
$146	8	R

Deterministic PDAs

PDA for LR(0) parsing is deterministic

Some CFLs require non-deterministic PDAs, such as

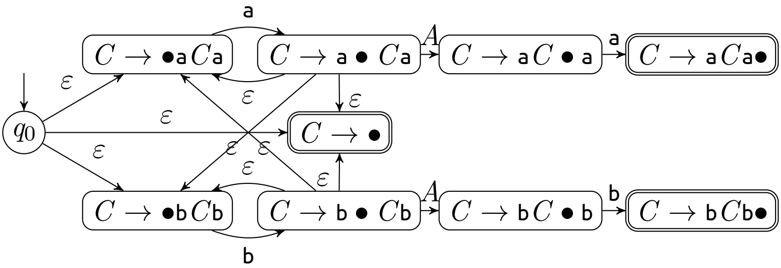
$$L = \{ww^R \mid w \in \{\mathsf{a}, \mathsf{b}\}^*\}$$

What goes wrong when we do LR(0) parsing on L?

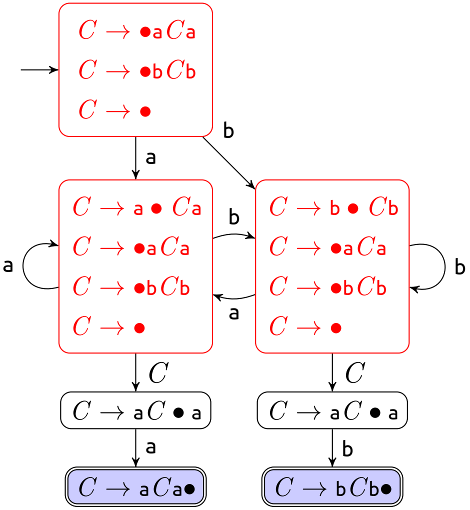
Example 2

$L = \{ww^R \mid w \in \{\mathsf{a}, \mathsf{b}\}^*\}$ $\qquad$ $C \to \mathsf{a}C\mathsf{a} \mid \mathsf{b}C\mathsf{b} \mid \varepsilon$

NFA N:



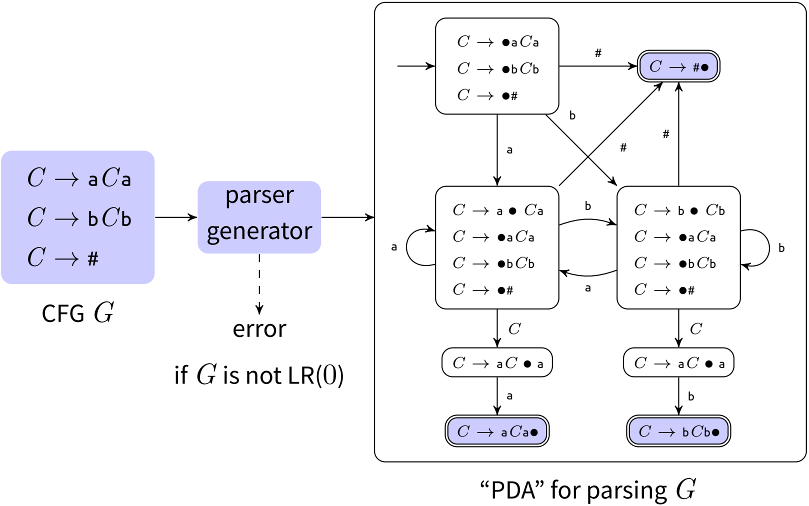
Example 2



$$C \to \mathsf{a} C \mathsf{a} \mid \mathsf{b} C \mathsf{b} \mid \varepsilon$$

shift-reduce conflicts

Parser generator



Motivation: Fast parsing for programming languages

LR(1) Grammar: A few words

LR(0) grammar revisited

LR(1) grammars

LR(0) grammars

LR(0) parser: **L**eft-to-right read, **R**ightmost derivation, $\mathbf{0}$ lookahead symbol

$$S \to SA \mid A$$
$$A \to \texttt{(}S\texttt{)} \mid \texttt{()}$$

Derivation

$$S \Rightarrow SA \Rightarrow S\texttt{()} \Rightarrow A\texttt{()} \Rightarrow \texttt{()()}$$

Reduction (derivation in reverse)

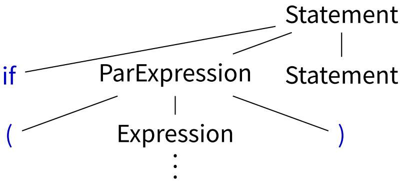
$$\texttt{()()} \rightarrowtail A\texttt{()} \rightarrowtail S\texttt{()} \rightarrowtail SA \rightarrowtail S$$

LR(0) parser looks for rightmost derivation

Rightmost derivation $=$ Leftmost reduction

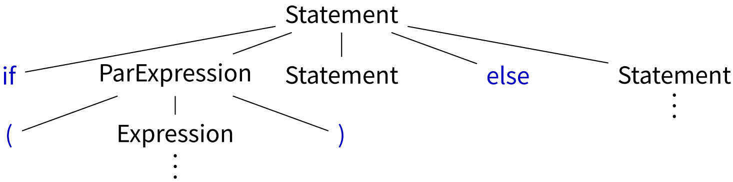
Parsing computer programs

```
if (n == 0) { return x; }
```



Parsing computer programs

```
if (n == 0) { return x; }
else { return x + 1; }
```



CFGs of most programming languages are not LR(0)

LR(0) parser cannot tell apart

`if ...then` from `if ...then ...else`

LR(1) grammar

LR(1) grammars resolve such conflicts by one symbol lookahead

States in NFA N

LR(0):	LR(1):
$A \to \alpha \bullet \beta$	$[A \to \alpha \bullet \beta, a]$

States in DFA D

LR(0):	LR(1):
no shift-reduce conflicts	some shift-reduce conflicts allowed
no reduce-reduce conflicts	some reduce-reduce conflicts allowed
	as long as can be resolved with
	lookahead symbol a

We won't cover LR(1) parser in this class; take CSCI 3180 for details